Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.4-Improper/50-1.2.4.2-d-x-^m-a-x^q+b-x^n+c-x^-2-n-q-^p

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 6:29am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [140]. This is test number [50].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (140)	0.00 (0)
Mathematica	99.29 (139)	0.71 (1)
Maple	97.14 (136)	2.86 (4)
Fricas	96.43 (135)	3.57 (5)
Giac	77.86 (109)	22.14 (31)
Mupad	51.43 (72)	48.57 (68)
Sympy	33.57 (47)	66.43 (93)
Maxima	17.14 (24)	82.86 (116)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

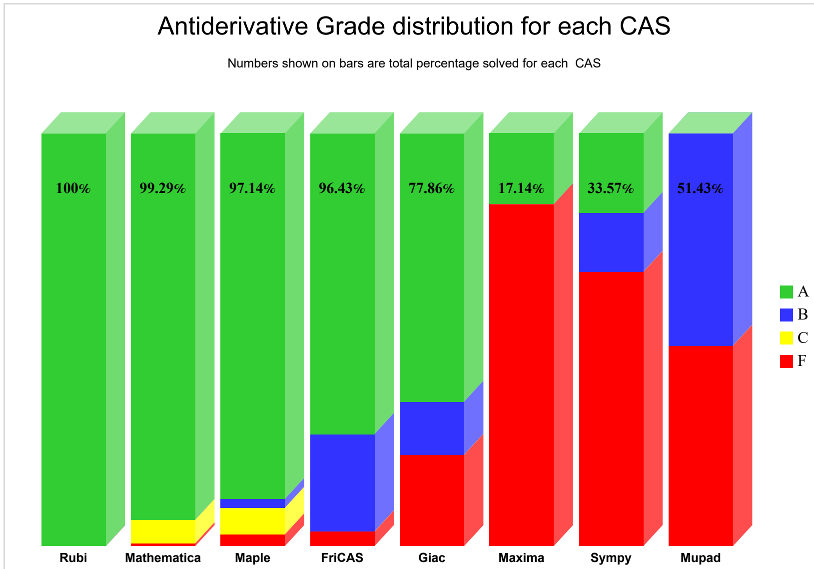
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

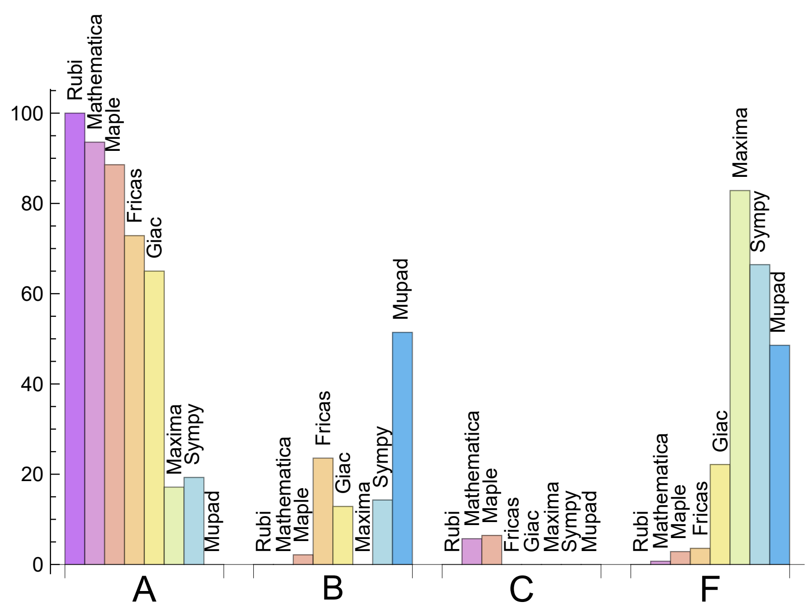
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	93.571	0.000	5.714	0.714
Maple	88.571	2.143	6.429	2.857
Fricas	72.857	23.571	0.000	3.571
Giac	65.000	12.857	0.000	22.143
Sympy	19.286	14.286	0.000	66.429
Maxima	17.143	0.000	0.000	82.857
Mupad	0.000	51.429	0.000	48.571

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Fricas	5	80.00	0.00	20.00
Giac	31	45.16	16.13	38.71
Mupad	68	0.00	100.00	0.00
Sympy	93	72.04	27.96	0.00
Maxima	116	81.90	0.86	17.24

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.19
Maple	0.26
Fricas	0.29
Rubi	0.33
Giac	0.39
Mathematica	0.73
Sympy	1.00
Mupad	4.78

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	32.71	0.81	28.50	0.81
Maple	133.00	0.95	96.00	0.85
Mathematica	136.34	1.02	108.00	0.98
Rubi	141.56	1.01	108.00	1.00
Sympy	199.02	2.69	124.00	0.94
Giac	358.72	2.07	79.00	1.09
Fricas	466.01	2.98	297.00	2.69
Mupad	1266.07	6.91	172.00	2.35

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

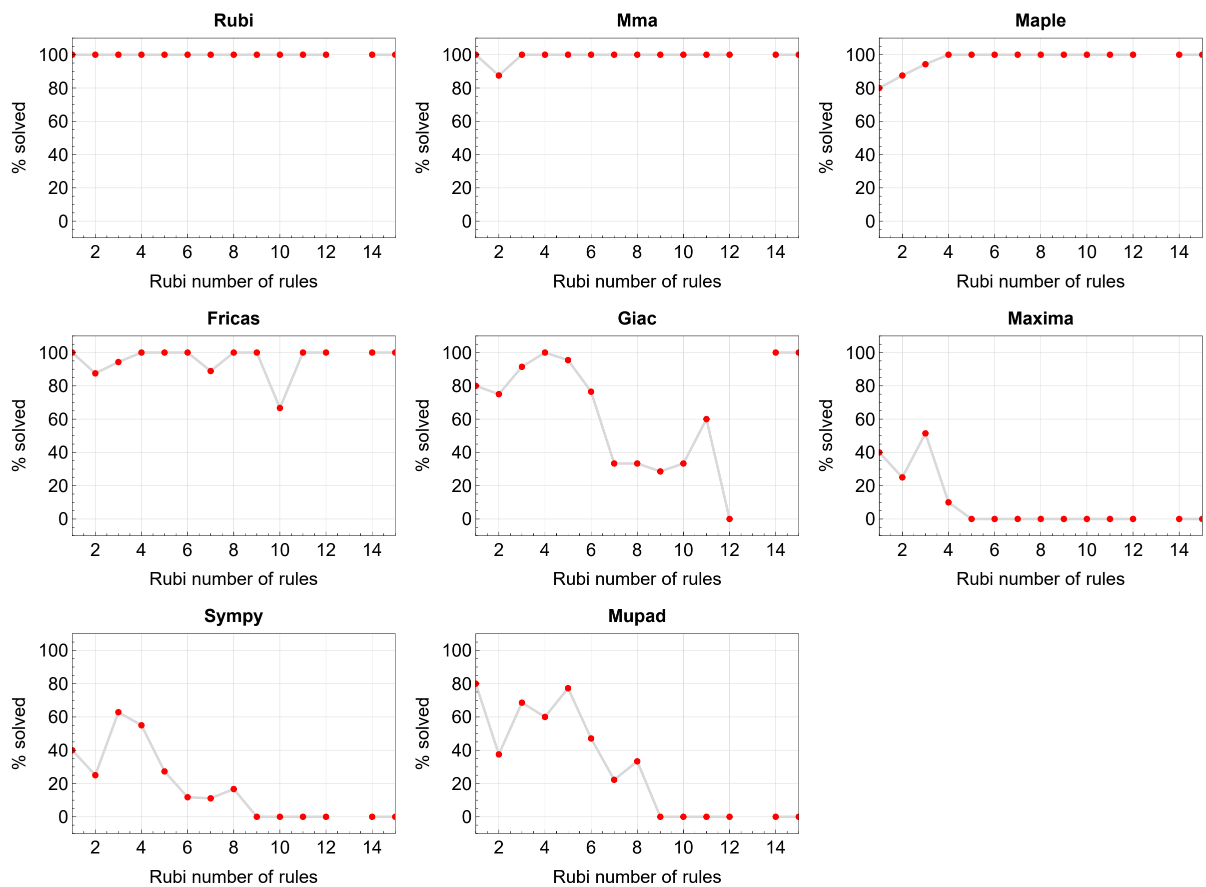


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

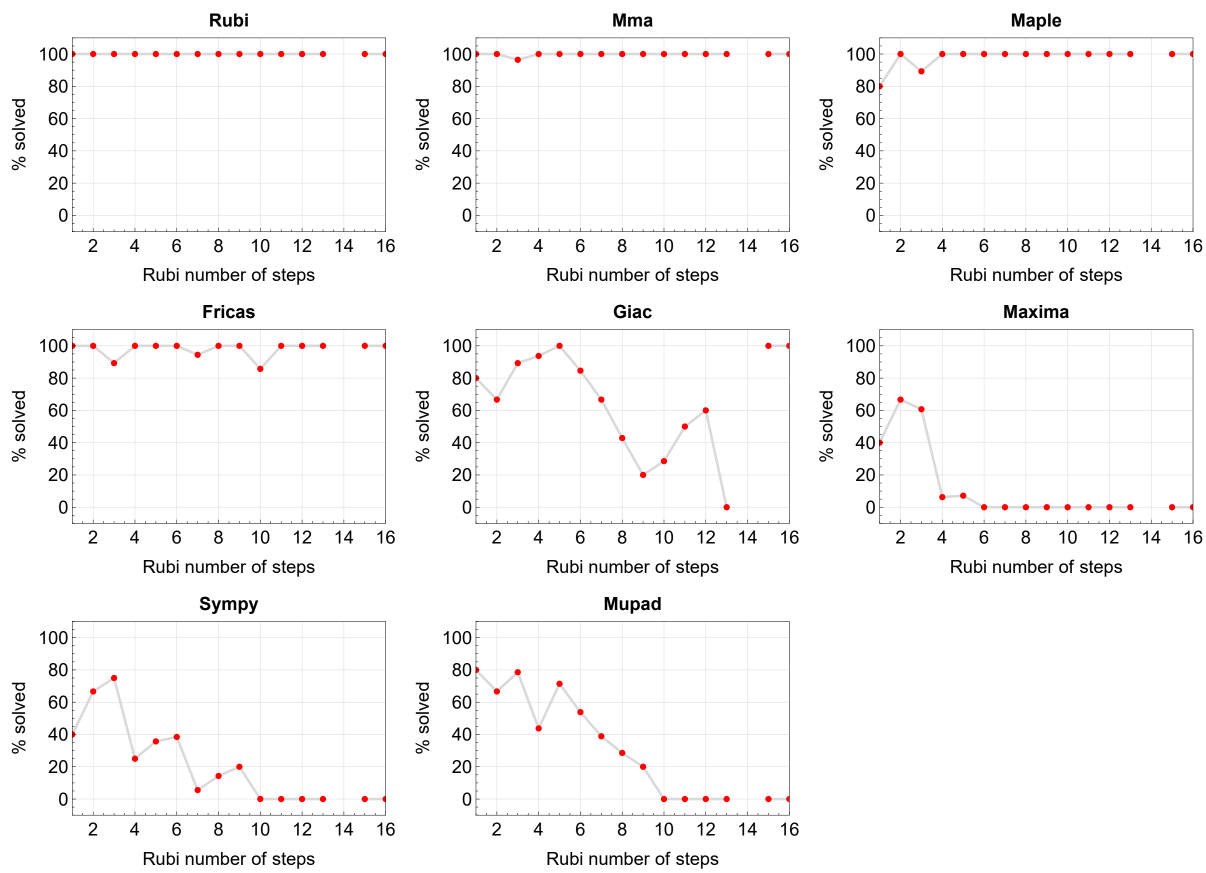


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

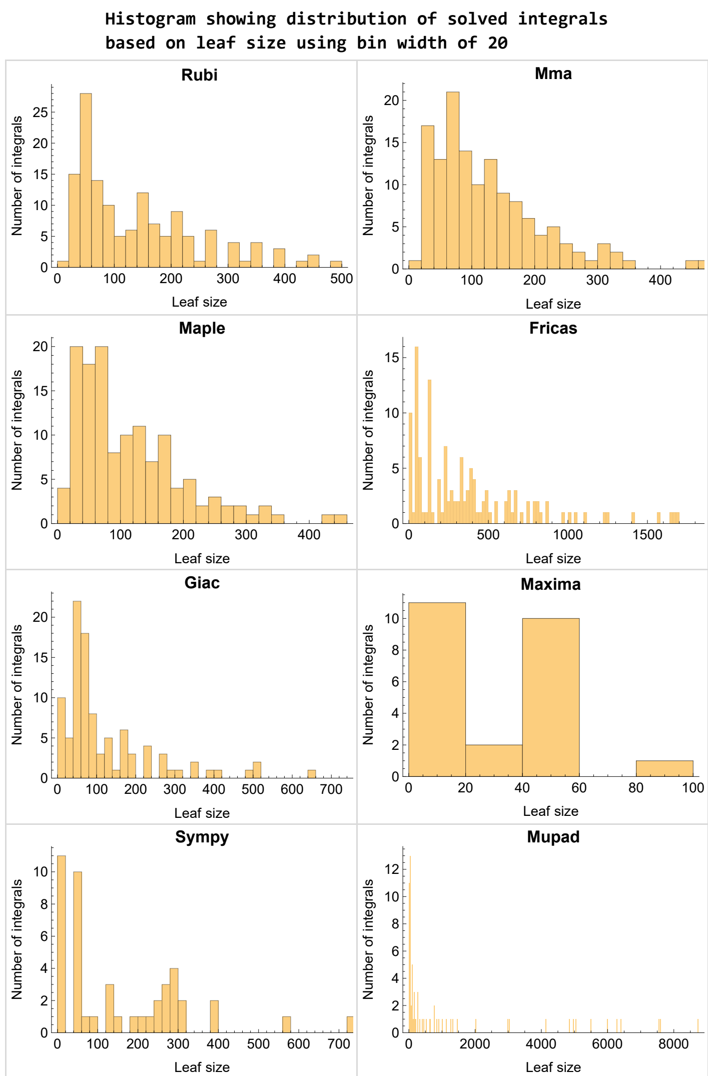


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

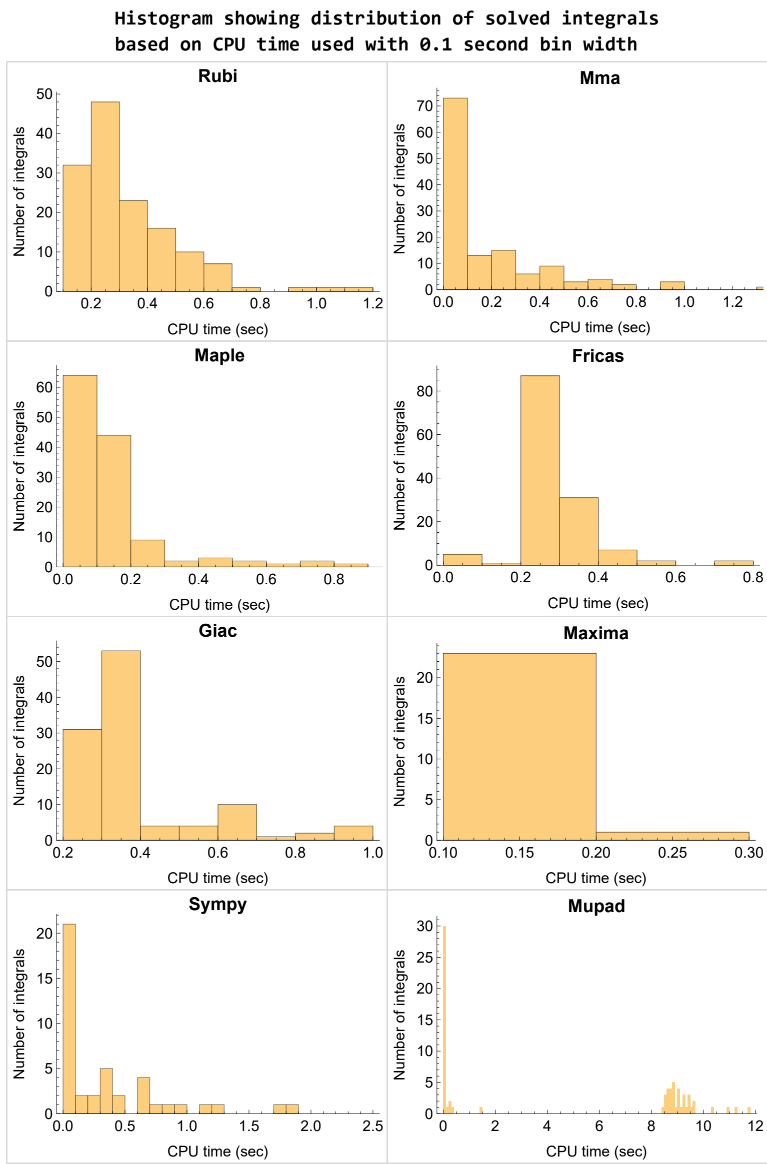


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

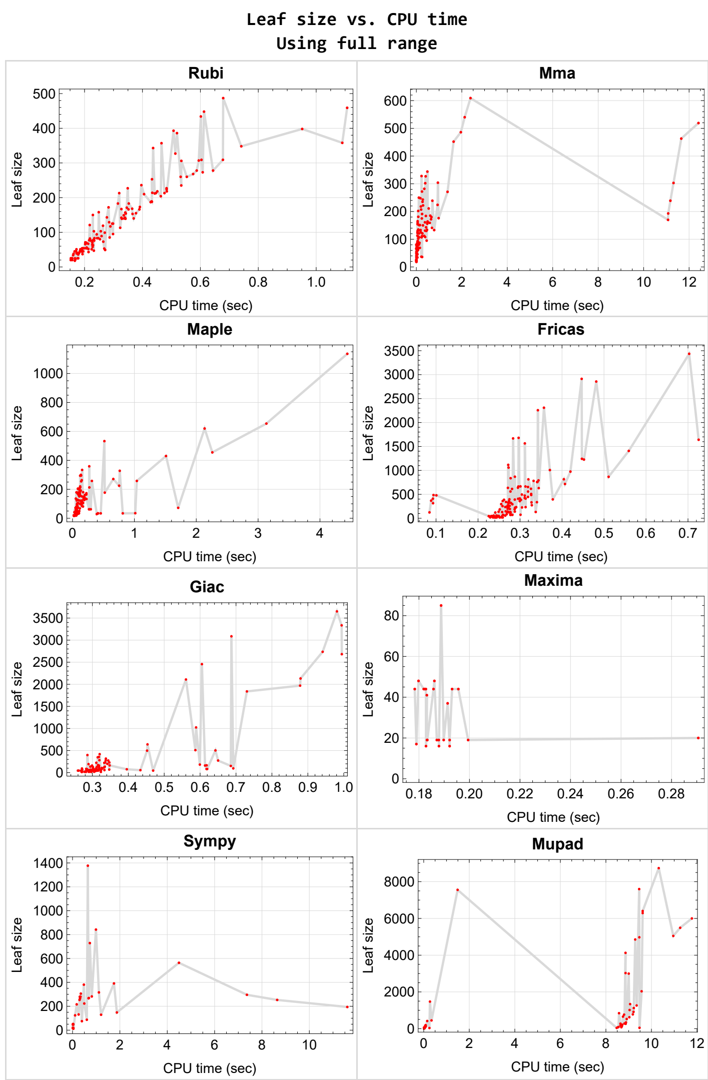


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

1.9. list of integrals with no known antiderivative

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

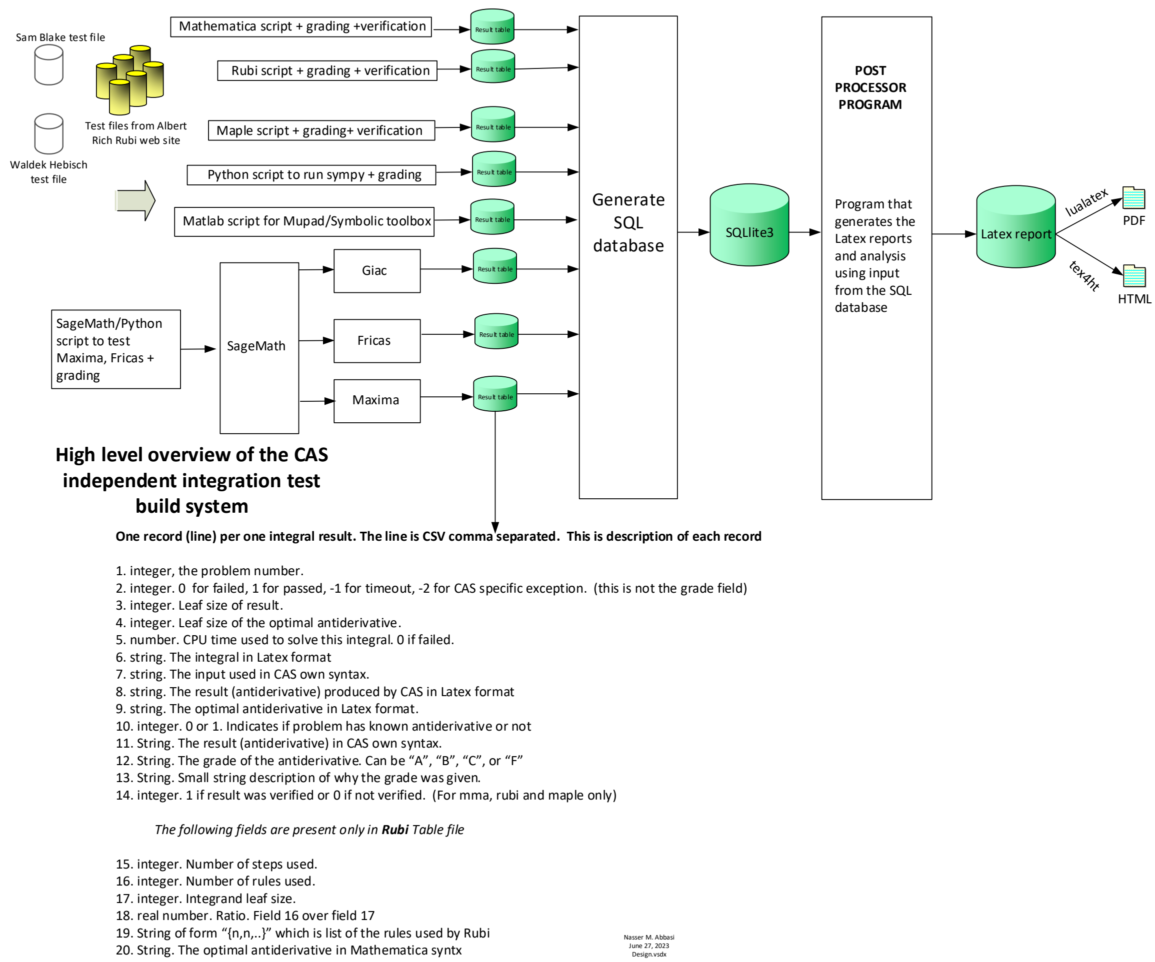
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 111, 113, 115, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { }

C grade { 105, 107, 110, 112, 114, 116, 117, 119 }

F normal fail { 140 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 87, 88, 89, 91, 93, 95, 97, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 72, 118, 119 }

C grade { 79, 81, 83, 85, 90, 92, 94, 96, 98 }

F normal fail { 104, 121, 122, 140 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 60, 72, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 118 }

C grade { }

F normal fail { 104, 116, 119, 122 }

F(-1) timedout fail { }

F(-2) exception fail { 140 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 137 }

B grade { }

C grade { }

F normal fail { 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 139, 140 }

F(-1) timedout fail { 34 }

F(-2) exception fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 129, 133 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 49, 50, 51, 52, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103, 106, 113, 115, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 60, 65, 72, 79, 81, 83, 85, 87, 90, 92, 94, 96, 98, 100, 102, 109, 111, 118 }

C grade { }

F normal fail { 34, 46, 104, 105, 107, 110, 112, 114, 116, 117, 119, 121, 122, 140 }

F(-1) timedout fail { 61, 62, 63, 64, 120 }

F(-2) exception fail { 33, 35, 36, 37, 43, 44, 45, 47, 48, 53, 54, 108 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 129, 133, 137 }

C grade { }

F normal fail { }

F(-1) timedout fail { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 139, 140 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 79, 81, 83, 85, 87, 94 }

B grade { 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 65, 72, 78, 80, 82, 84, 86, 93, 95, 97 }

C grade { }

F normal fail { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 137, 138, 140 }

F(-1) timedout fail { 16, 17, 18, 24, 25, 26, 27, 28, 88, 89, 90, 91, 92, 96, 98, 99, 100, 101, 102, 103, 109, 131, 132, 135, 136, 139 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.156	0.004	0.054	0.188	0.229	0.018	0.292	0.018

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.160	0.005	0.054	0.199	0.235	0.023	0.297	0.017

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.147	0.000	0.054	0.192	0.237	0.026	0.278	0.017

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.161	0.002	0.062	0.190	0.245	0.018	0.289	0.018

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.151	0.007	0.023	0.192	0.241	0.018	0.324	0.014

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83
time (sec)	N/A	0.203	0.010	0.084	0.183	0.226	0.021	0.308	0.020

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83
time (sec)	N/A	0.200	0.008	0.083	0.193	0.231	0.020	0.316	0.014

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.89	0.81	0.89	0.85	0.83
time (sec)	N/A	0.207	0.009	0.059	0.186	0.232	0.021	0.293	0.013

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83
time (sec)	N/A	0.200	0.010	0.078	0.196	0.226	0.023	0.313	0.014

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83
time (sec)	N/A	0.195	0.008	0.078	0.186	0.237	0.023	0.288	0.014

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.262	0.074	0.076	0.000	0.255	0.478	0.283	0.079

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.230	0.041	0.070	0.000	0.265	0.350	0.295	8.643

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.204	0.024	0.051	0.000	0.252	0.176	0.308	0.075

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35
time (sec)	N/A	0.160	0.006	0.045	0.000	0.241	0.108	0.309	0.019

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	63	61	61	0	211	564	62	213
N.S.	1	1.02	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.227	0.044	0.061	0.000	0.273	4.494	0.303	8.719

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	85	77	81	0	269	0	79	339
N.S.	1	1.05	0.95	1.00	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.284	0.056	0.063	0.000	0.271	0.000	0.285	8.882

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	113	102	128	0	358	0	105	447
N.S.	1	1.09	0.98	1.23	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.319	0.084	0.082	0.000	0.299	0.000	0.326	0.342

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	140	131	157	0	445	0	136	524
N.S.	1	1.02	0.96	1.15	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.374	0.070	0.124	0.000	0.319	0.000	0.294	8.914

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	154	132	198	0	837	842	161	261
N.S.	1	1.03	0.88	1.32	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.354	0.143	0.107	0.000	0.278	0.989	0.299	8.869

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	125	109	169	0	635	729	125	279
N.S.	1	1.10	0.96	1.48	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.302	0.097	0.119	0.000	0.271	0.731	0.302	8.815

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	387	280	88	135
N.S.	1	1.00	1.21	1.45	0.00	5.78	4.18	1.31	2.01
time (sec)	N/A	0.205	0.119	0.071	0.000	0.262	0.307	0.302	8.703

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.205	0.045	0.067	0.000	0.274	0.295	0.396	8.685

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.201	0.050	0.065	0.000	0.267	0.318	0.305	0.050

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	139	107	177	0	781	0	126	620
N.S.	1	1.29	0.99	1.64	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.343	0.115	0.089	0.000	0.332	0.000	0.312	9.029

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	173	131	205	0	975	0	171	775
N.S.	1	1.17	0.89	1.39	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.393	0.384	0.087	0.000	0.420	0.000	0.316	9.183

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	213	175	255	0	1226	0	229	914
N.S.	1	1.05	0.87	1.26	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.450	0.241	0.140	0.000	0.452	0.000	0.336	9.214

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	260	218	295	0	1407	0	282	1120
N.S.	1	1.03	0.87	1.17	0.00	5.58	0.00	1.12	4.44
time (sec)	N/A	0.538	0.199	0.139	0.000	0.559	0.000	0.319	9.225

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	309	272	359	0	1640	0	347	1260
N.S.	1	0.97	0.86	1.13	0.00	5.16	0.00	1.09	3.96
time (sec)	N/A	0.602	0.248	0.270	0.000	0.725	0.000	0.315	9.350

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	278	184	133	0	390	0	275	0
N.S.	1	1.08	0.72	0.52	0.00	1.52	0.00	1.07	0.00
time (sec)	N/A	0.652	0.419	0.226	0.000	0.288	0.000	0.344	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	218	150	106	0	326	0	222	0
N.S.	1	1.06	0.73	0.52	0.00	1.59	0.00	1.08	0.00
time (sec)	N/A	0.474	0.291	0.171	0.000	0.272	0.000	0.333	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	143	121	121	0	260	0	160	0
N.S.	1	0.88	0.74	0.74	0.00	1.60	0.00	0.98	0.00
time (sec)	N/A	0.270	0.265	0.130	0.000	0.275	0.000	0.318	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	95	100	0	220	0	119	0
N.S.	1	1.00	0.80	0.84	0.00	1.85	0.00	1.00	0.00
time (sec)	N/A	0.266	0.453	0.118	0.000	0.288	0.000	0.333	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	140	133	95	0	638	0	0	0
N.S.	1	0.81	0.77	0.55	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	0.343	0.186	0.141	0.000	0.295	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	142	125	101	0	653	0	0	0
N.S.	1	0.82	0.72	0.58	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.334	0.199	0.128	0.000	0.319	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	117	111	81	0	226	0	0	0
N.S.	1	1.03	0.97	0.71	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.284	0.256	0.169	0.000	0.305	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	166	130	115	0	272	0	0	0
N.S.	1	1.07	0.84	0.74	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.379	0.440	0.154	0.000	0.328	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	224	159	149	0	336	0	0	0
N.S.	1	1.09	0.78	0.73	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.485	0.590	0.189	0.000	0.340	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	459	304	328	0	664	0	509	0
N.S.	1	1.09	0.72	0.78	0.00	1.57	0.00	1.21	0.00
time (sec)	N/A	1.098	0.948	0.763	0.000	0.313	0.000	0.587	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	398	239	271	0	558	0	419	0
N.S.	1	1.09	0.66	0.74	0.00	1.53	0.00	1.15	0.00
time (sec)	N/A	0.973	0.705	0.659	0.000	0.319	0.000	0.320	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	309	211	225	0	474	0	355	0
N.S.	1	1.07	0.73	0.78	0.00	1.65	0.00	1.23	0.00
time (sec)	N/A	0.675	0.543	0.753	0.000	0.311	0.000	0.318	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	210	161	126	0	384	0	276	0
N.S.	1	1.06	0.81	0.64	0.00	1.94	0.00	1.39	0.00
time (sec)	N/A	0.403	0.378	0.191	0.000	0.302	0.000	0.337	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	171	133	100	0	320	0	224	0
N.S.	1	1.04	0.81	0.61	0.00	1.94	0.00	1.36	0.00
time (sec)	N/A	0.338	0.785	0.153	0.000	0.278	0.000	0.311	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	204	164	192	0	791	0	0	0
N.S.	1	0.90	0.72	0.85	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.467	0.495	0.133	0.000	0.344	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	189	156	180	0	757	0	0	0
N.S.	1	0.86	0.71	0.82	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.446	0.480	0.171	0.000	0.341	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	188	160	140	0	757	0	0	0
N.S.	1	0.86	0.73	0.64	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.425	0.502	0.179	0.000	0.341	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	235	172	150	0	815	0	0	0
N.S.	1	0.91	0.67	0.58	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.543	0.646	0.187	0.000	0.404	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	213	141	131	0	332	0	0	0
N.S.	1	1.08	0.72	0.66	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.473	0.687	0.178	0.000	0.327	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	273	176	161	0	394	0	0	0
N.S.	1	1.10	0.71	0.65	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.612	0.980	0.199	0.000	0.378	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	149	107	100	0	226	0	134	0
N.S.	1	1.04	0.75	0.70	0.00	1.58	0.00	0.94	0.00
time (sec)	N/A	0.332	0.229	0.139	0.000	0.290	0.000	0.335	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	89	50	0	188	0	90	0
N.S.	1	1.00	0.86	0.49	0.00	1.83	0.00	0.87	0.00
time (sec)	N/A	0.257	0.045	0.109	0.000	0.287	0.000	0.319	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	29	0	129	0	57	0
N.S.	1	1.00	0.94	0.41	0.00	1.82	0.00	0.80	0.00
time (sec)	N/A	0.201	0.079	0.081	0.000	0.273	0.000	0.334	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	68	42	0	130	0	59	0
N.S.	1	1.00	1.51	0.93	0.00	2.89	0.00	1.31	0.00
time (sec)	N/A	0.170	0.081	0.100	0.000	0.277	0.000	0.324	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	87	68	0	194	0	0	0
N.S.	1	1.00	1.13	0.88	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.220	0.141	0.118	0.000	0.276	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	125	111	117	0	232	0	0	0
N.S.	1	1.05	0.93	0.98	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.296	0.247	0.143	0.000	0.313	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	268	185	164	0	616	0	318	0
N.S.	1	1.02	0.71	0.63	0.00	2.35	0.00	1.21	0.00
time (sec)	N/A	0.581	0.611	0.224	0.000	0.326	0.000	0.338	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	212	143	129	0	486	0	231	0
N.S.	1	1.05	0.71	0.64	0.00	2.42	0.00	1.15	0.00
time (sec)	N/A	0.444	0.428	0.189	0.000	0.327	0.000	0.347	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	166	112	84	0	414	0	164	0
N.S.	1	1.08	0.73	0.55	0.00	2.71	0.00	1.07	0.00
time (sec)	N/A	0.340	0.347	0.153	0.000	0.300	0.000	0.348	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	34	0	73	0	69	75
N.S.	1	1.00	0.92	0.85	0.00	1.82	0.00	1.72	1.88
time (sec)	N/A	0.172	0.219	0.104	0.000	0.301	0.000	0.304	8.679

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	33	0	72	0	74	75
N.S.	1	1.00	0.92	0.85	0.00	1.85	0.00	1.90	1.92
time (sec)	N/A	0.170	0.255	0.098	0.000	0.277	0.000	0.304	8.526

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	108	107	0	411	0	199	0
N.S.	1	1.00	1.15	1.14	0.00	4.37	0.00	2.12	0.00
time (sec)	N/A	0.243	0.387	0.151	0.000	0.313	0.000	0.342	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	157	134	137	0	496	0	0	0
N.S.	1	1.09	0.93	0.95	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	0.338	0.470	0.204	0.000	0.304	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	218	180	173	0	630	0	0	0
N.S.	1	1.04	0.86	0.83	0.00	3.01	0.00	0.00	0.00
time (sec)	N/A	0.443	0.663	0.224	0.000	0.345	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	278	224	213	0	716	0	0	0
N.S.	1	1.03	0.83	0.79	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	0.588	0.933	0.272	0.000	0.407	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	348	271	258	0	866	0	0	0
N.S.	1	1.01	0.79	0.75	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.742	1.372	0.313	0.000	0.511	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	47	37	71	280	107	89
N.S.	1	1.00	0.92	1.27	1.00	1.92	7.57	2.89	2.41
time (sec)	N/A	0.190	0.045	0.060	0.191	0.282	0.335	0.311	8.552

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.170	0.002	0.058	0.183	0.243	0.018	0.305	0.017

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.171	0.002	0.020	0.183	0.251	0.017	0.271	0.017

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.155	0.000	0.021	0.187	0.252	0.018	0.271	0.016

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.158	0.001	0.023	0.183	0.242	0.018	0.282	0.014

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.95	0.81
time (sec)	N/A	0.170	0.002	0.053	0.179	0.259	0.035	0.270	0.015

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	12	16	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89
time (sec)	N/A	0.168	0.002	0.029	0.188	0.256	0.037	0.271	0.017

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	300	85	241	1377	399	271
N.S.	1	1.00	0.91	3.95	1.12	3.17	18.12	5.25	3.57
time (sec)	N/A	0.236	0.155	0.135	0.189	0.268	0.646	0.286	8.640

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.85	0.83
time (sec)	N/A	0.207	0.006	0.095	0.182	0.234	0.020	0.260	0.019

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	58	48	45	44	44	46	46	45
N.S.	1	1.07	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.227	0.007	0.094	0.182	0.235	0.022	0.262	0.013

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	44	51	46	45
N.S.	1	1.00	1.00	0.83	0.89	0.81	0.94	0.85	0.83
time (sec)	N/A	0.209	0.006	0.060	0.180	0.249	0.024	0.269	0.013

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	48	45	44	44	46	46	45
N.S.	1	0.96	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.212	0.006	0.081	0.178	0.257	0.022	0.469	0.014

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	41	41	48	43	42
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.98	0.88	0.86
time (sec)	N/A	0.193	0.004	0.030	0.183	0.245	0.022	0.270	0.013

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	99	93	105	0	313	391	92	842
N.S.	1	0.99	0.93	1.05	0.00	3.13	3.91	0.92	8.42
time (sec)	N/A	0.281	0.063	0.091	0.000	0.273	1.750	0.327	8.576

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	214	250	73	0	1564	194	2457	4127
N.S.	1	1.05	1.23	0.36	0.00	7.70	0.96	12.10	20.33
time (sec)	N/A	0.443	0.110	0.085	0.000	0.311	11.603	0.605	8.862

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	80	78	83	0	254	316	75	655
N.S.	1	0.99	0.96	1.02	0.00	3.14	3.90	0.93	8.09
time (sec)	N/A	0.256	0.030	0.078	0.000	0.258	1.114	0.309	8.751

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	183	202	57	0	1059	129	2109	3026
N.S.	1	1.02	1.13	0.32	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.335	0.083	0.069	0.000	0.273	1.205	0.561	8.863

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	62	60	0	197	223	59	118
N.S.	1	1.02	0.98	0.95	0.00	3.13	3.54	0.94	1.87
time (sec)	N/A	0.231	0.016	0.053	0.000	0.271	0.496	0.284	0.092

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	158	165	41	0	559	75	503	416
N.S.	1	1.05	1.10	0.27	0.00	3.73	0.50	3.35	2.77
time (sec)	N/A	0.269	0.057	0.069	0.000	0.277	0.395	0.643	0.153

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	129	131	35	41
N.S.	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14
time (sec)	N/A	0.184	0.008	0.040	0.000	0.269	0.257	0.282	8.474

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	38	0	613	87	1024	763
N.S.	1	1.00	0.86	0.25	0.00	4.09	0.58	6.83	5.09
time (sec)	N/A	0.243	0.053	0.064	0.000	0.281	0.603	0.589	8.799

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	73	113	65	0	223	253	68	1014
N.S.	1	1.06	1.64	0.94	0.00	3.23	3.67	0.99	14.70
time (sec)	N/A	0.253	0.046	0.059	0.000	0.272	8.642	0.272	9.041

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	172	191	159	0	1116	148	1839	2997
N.S.	1	0.99	1.10	0.91	0.00	6.41	0.85	10.57	17.22
time (sec)	N/A	0.307	0.253	0.099	0.000	0.272	1.873	0.730	9.003

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	95	135	85	0	293	0	94	2033
N.S.	1	1.07	1.52	0.96	0.00	3.29	0.00	1.06	22.84
time (sec)	N/A	0.300	0.084	0.079	0.000	0.282	0.000	0.271	9.569

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	168	151	209	0	868	0	161	1473
N.S.	1	1.01	0.91	1.26	0.00	5.23	0.00	0.97	8.87
time (sec)	N/A	0.371	0.120	0.116	0.000	0.288	0.000	0.614	0.274

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	306	327	174	0	2856	0	3335	7599
N.S.	1	0.92	0.99	0.53	0.00	8.63	0.00	10.08	22.96
time (sec)	N/A	0.559	0.409	0.105	0.000	0.482	0.000	0.994	9.461

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	139	121	179	0	663	0	152	1336
N.S.	1	1.05	0.92	1.36	0.00	5.02	0.00	1.15	10.12
time (sec)	N/A	0.331	0.118	0.100	0.000	0.296	0.000	0.685	9.064

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	253	282	151	0	2257	0	2736	6293
N.S.	1	0.93	1.04	0.56	0.00	8.33	0.00	10.10	23.22
time (sec)	N/A	0.463	0.350	0.109	0.000	0.343	0.000	0.941	9.614

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	93	104	0	407	282	96	187
N.S.	1	1.01	1.19	1.33	0.00	5.22	3.62	1.23	2.40
time (sec)	N/A	0.227	0.059	0.078	0.000	0.296	0.813	0.692	0.087

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	227	235	123	0	1668	296	2132	4973
N.S.	1	0.96	0.99	0.52	0.00	7.04	1.25	9.00	20.98
time (sec)	N/A	0.372	0.272	0.106	0.000	0.284	7.364	0.880	9.469

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	76	79	77	0	360	269	82	178
N.S.	1	1.01	1.05	1.03	0.00	4.80	3.59	1.09	2.37
time (sec)	N/A	0.230	0.046	0.079	0.000	0.274	0.690	0.620	0.082

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	213	222	122	0	1680	0	1970	4854
N.S.	1	0.96	1.00	0.55	0.00	7.60	0.00	8.91	21.96
time (sec)	N/A	0.337	0.295	0.148	0.000	0.297	0.000	0.878	9.288

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	76	79	75	0	361	267	82	172
N.S.	1	1.03	1.07	1.01	0.00	4.88	3.61	1.11	2.32
time (sec)	N/A	0.224	0.054	0.084	0.000	0.259	0.675	0.618	0.074

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	236	243	151	0	2309	0	2682	6404
N.S.	1	0.94	0.96	0.60	0.00	9.16	0.00	10.64	25.41
time (sec)	N/A	0.415	0.283	0.150	0.000	0.357	0.000	0.995	9.615

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	153	207	185	0	813	0	166	5048
N.S.	1	1.25	1.70	1.52	0.00	6.66	0.00	1.36	41.38
time (sec)	N/A	0.371	0.209	0.126	0.000	0.320	0.000	0.618	10.960

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	307	302	294	0	2912	0	3087	7555
N.S.	1	1.00	0.98	0.95	0.00	9.45	0.00	10.02	24.53
time (sec)	N/A	0.628	0.391	0.123	0.000	0.447	0.000	0.687	1.488

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	187	248	213	0	1007	0	182	5491
N.S.	1	1.15	1.53	1.31	0.00	6.22	0.00	1.12	33.90
time (sec)	N/A	0.430	0.177	0.124	0.000	0.371	0.000	0.599	11.262

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	358	344	334	0	3435	0	3651	8739
N.S.	1	0.99	0.95	0.93	0.00	9.52	0.00	10.11	24.21
time (sec)	N/A	1.109	0.485	0.155	0.000	0.703	0.000	0.981	10.321

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	227	328	263	0	1242	0	274	5999
N.S.	1	1.04	1.50	1.20	0.00	5.67	0.00	1.25	27.39
time (sec)	N/A	0.474	0.228	0.149	0.000	0.447	0.000	0.650	11.776

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	170	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	11.074	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	386	486	455	0	368	0	0	0
N.S.	1	1.02	1.28	1.20	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.540	1.958	2.260	0.000	0.088	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	108	123	0	232	0	121	0
N.S.	1	1.00	0.84	0.95	0.00	1.80	0.00	0.94	0.00
time (sec)	N/A	0.285	0.091	0.067	0.000	0.291	0.000	0.319	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	357	452	430	0	314	0	0	0
N.S.	1	1.03	1.30	1.24	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.461	1.639	1.511	0.000	0.093	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	155	154	136	0	666	0	0	0
N.S.	1	0.80	0.79	0.70	0.00	3.43	0.00	0.00	0.00
time (sec)	N/A	0.375	0.055	0.053	0.000	0.301	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	260	181	202	0	396	0	640	0
N.S.	1	1.07	0.74	0.83	0.00	1.62	0.00	2.62	0.00
time (sec)	N/A	0.552	0.086	0.090	0.000	0.275	0.000	0.453	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	609	654	0	485	0	0	0
N.S.	1	1.00	1.25	1.34	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.692	2.382	3.134	0.000	0.094	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	183	151	170	0	332	0	498	0
N.S.	1	1.03	0.85	0.96	0.00	1.88	0.00	2.81	0.00
time (sec)	N/A	0.349	0.159	0.082	0.000	0.274	0.000	0.452	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	434	540	620	0	409	0	0	0
N.S.	1	1.02	1.27	1.46	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.612	2.127	2.135	0.000	0.093	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	72	0	135	0	56	0
N.S.	1	1.00	0.98	0.88	0.00	1.65	0.00	0.68	0.00
time (sec)	N/A	0.220	0.013	0.048	0.000	0.270	0.000	0.311	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	193	177	0	123	0	0	0
N.S.	1	1.00	1.60	1.46	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.225	11.081	0.521	0.000	0.084	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	80	72	0	137	0	56	0
N.S.	1	1.00	1.57	1.41	0.00	2.69	0.00	1.10	0.00
time (sec)	N/A	0.175	0.018	0.053	0.000	0.249	0.000	0.434	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	343	303	258	0	0	0	0	0
N.S.	1	1.04	0.92	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	11.314	1.039	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	393	463	533	0	479	0	0	0
N.S.	1	1.01	1.18	1.36	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.494	11.656	0.515	0.000	0.101	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	123	179	0	424	0	193	0
N.S.	1	1.00	1.19	1.74	0.00	4.12	0.00	1.87	0.00
time (sec)	N/A	0.237	0.047	0.063	0.000	0.311	0.000	0.289	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	448	519	1136	0	0	0	0	0
N.S.	1	0.96	1.11	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	12.413	4.442	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	167	159	220	0	508	0	0	0
N.S.	1	1.08	1.03	1.43	0.00	3.30	0.00	0.00	0.00
time (sec)	N/A	0.330	0.057	0.123	0.000	0.327	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	46	0	0	83	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.186	0.048	0.000	0.000	0.268	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	327	239	0	0	0	0	0	0
N.S.	1	1.14	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	11.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.169	0.087	0.812	0.000	0.244	0.000	0.316	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.186	0.001	0.456	0.000	0.245	0.000	0.315	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.186	0.001	0.410	0.000	0.265	0.000	0.308	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	83	80	62	0	70	0	69	0
N.S.	1	0.97	0.93	0.72	0.00	0.81	0.00	0.80	0.00
time (sec)	N/A	0.234	0.071	0.295	0.000	0.259	0.000	0.314	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	83	80	62	0	70	0	69	0
N.S.	1	0.97	0.93	0.72	0.00	0.81	0.00	0.80	0.00
time (sec)	N/A	0.254	0.001	0.271	0.000	0.260	0.000	0.311	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	83	80	62	0	70	0	69	0
N.S.	1	0.97	0.93	0.72	0.00	0.81	0.00	0.80	0.00
time (sec)	N/A	0.248	0.001	0.266	0.000	0.278	0.000	0.347	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	0	111	0	35	34
N.S.	1	1.00	0.97	0.92	0.00	2.92	0.00	0.92	0.89
time (sec)	N/A	0.165	0.011	1.014	0.000	0.277	0.000	0.298	0.045

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	68	42	0	130	0	59	0
N.S.	1	1.00	1.51	0.93	0.00	2.89	0.00	1.31	0.00
time (sec)	N/A	0.195	0.013	0.089	0.000	0.257	0.000	0.308	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	70	64	0	131	0	53	0
N.S.	1	1.00	1.49	1.36	0.00	2.79	0.00	1.13	0.00
time (sec)	N/A	0.231	0.021	0.067	0.000	0.337	0.000	0.318	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	72	66	0	139	0	58	0
N.S.	1	1.00	1.47	1.35	0.00	2.84	0.00	1.18	0.00
time (sec)	N/A	0.273	0.018	0.064	0.000	0.288	0.000	0.290	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	0	124	0	38	44
N.S.	1	1.00	0.93	0.89	0.00	2.82	0.00	0.86	1.00
time (sec)	N/A	0.182	0.005	0.071	0.000	0.273	0.000	0.296	9.476

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	72	0	135	0	62	0
N.S.	1	1.00	1.59	1.47	0.00	2.76	0.00	1.27	0.00
time (sec)	N/A	0.188	0.011	1.708	0.000	0.271	0.000	0.308	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	80	72	0	137	0	56	0
N.S.	1	1.00	1.57	1.41	0.00	2.69	0.00	1.10	0.00
time (sec)	N/A	0.222	0.011	0.052	0.000	0.271	0.000	0.284	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	82	74	0	145	0	61	0
N.S.	1	1.00	1.55	1.40	0.00	2.74	0.00	1.15	0.00
time (sec)	N/A	0.260	0.013	0.074	0.000	0.266	0.000	0.299	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	33	29	20	47	0	55	33
N.S.	1	1.00	0.82	0.72	0.50	1.18	0.00	1.38	0.82
time (sec)	N/A	0.174	0.003	0.392	0.291	0.245	0.000	0.300	0.251

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	66	34	0	55	0	60	0
N.S.	1	1.00	1.47	0.76	0.00	1.22	0.00	1.33	0.00
time (sec)	N/A	0.182	0.002	0.412	0.000	0.237	0.000	0.315	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	50	0	49	0	47	0
N.S.	1	1.00	1.42	1.16	0.00	1.14	0.00	1.09	0.00
time (sec)	N/A	0.211	0.074	0.071	0.000	0.249	0.000	0.302	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [39] had the largest ratio of [.6999999999999999956]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	20	0.150
2	A	3	3	1.00	18	0.167
3	A	1	1	1.00	16	0.062
4	A	3	3	1.00	20	0.150
5	A	2	2	1.00	20	0.100
6	A	3	3	1.00	22	0.136
7	A	3	3	1.00	20	0.150
8	A	3	3	1.00	18	0.167
9	A	3	3	1.00	22	0.136
10	A	3	3	1.00	22	0.136
11	A	3	3	1.00	22	0.136
12	A	3	3	1.00	22	0.136
13	A	6	5	1.00	22	0.227
14	A	4	3	1.00	22	0.136
15	A	8	7	1.02	20	0.350
16	A	5	5	1.05	18	0.278
17	A	5	5	1.09	22	0.227
18	A	5	5	1.02	22	0.227
19	A	5	5	1.03	22	0.227
20	A	4	4	1.10	22	0.182
21	A	5	4	1.00	22	0.182
22	A	5	4	1.00	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	5	4	1.00	22	0.182
24	A	5	5	1.29	22	0.227
25	A	5	5	1.17	22	0.227
26	A	5	5	1.05	20	0.250
27	A	5	5	1.03	18	0.278
28	A	5	5	0.97	22	0.227
29	A	12	11	1.08	24	0.458
30	A	10	9	1.06	22	0.409
31	A	6	5	0.88	20	0.250
32	A	5	4	1.00	24	0.167
33	A	8	7	0.81	24	0.292
34	A	8	7	0.82	24	0.292
35	A	6	5	1.03	24	0.208
36	A	8	7	1.07	24	0.292
37	A	10	9	1.09	24	0.375
38	A	16	15	1.09	22	0.682
39	A	15	14	1.09	20	0.700
40	A	12	11	1.07	24	0.458
41	A	7	6	1.06	24	0.250
42	A	6	5	1.04	24	0.208
43	A	10	9	0.90	24	0.375
44	A	10	9	0.86	24	0.375
45	A	10	9	0.86	24	0.375
46	A	12	11	0.91	24	0.458
47	A	9	8	1.08	24	0.333
48	A	12	11	1.10	24	0.458
49	A	8	7	1.04	24	0.292
50	A	5	4	1.00	24	0.167
51	A	4	3	1.00	22	0.136
52	A	3	2	1.00	20	0.100
53	A	4	3	1.00	24	0.125
54	A	7	6	1.05	24	0.250
55	A	12	11	1.02	24	0.458
56	A	10	9	1.05	24	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	6	1.08	24	0.250
58	A	1	1	1.00	24	0.042
59	A	1	1	1.00	24	0.042
60	A	4	3	1.00	24	0.125
61	A	7	6	1.09	22	0.273
62	A	9	8	1.04	20	0.400
63	A	11	10	1.03	24	0.417
64	A	13	12	1.01	24	0.500
65	A	3	3	1.00	18	0.167
66	A	3	3	1.00	18	0.167
67	A	3	3	1.00	16	0.188
68	A	1	1	1.00	14	0.071
69	A	2	2	1.00	18	0.111
70	A	3	3	1.00	18	0.167
71	A	3	3	1.00	18	0.167
72	A	3	3	1.00	20	0.150
73	A	3	3	1.00	20	0.150
74	A	5	4	1.07	18	0.222
75	A	3	3	1.00	16	0.188
76	A	5	4	0.96	20	0.200
77	A	3	3	1.00	20	0.150
78	A	5	4	0.99	20	0.200
79	A	6	6	1.05	20	0.300
80	A	5	4	0.99	20	0.200
81	A	4	4	1.02	20	0.200
82	A	7	6	1.02	20	0.300
83	A	3	3	1.05	20	0.150
84	A	5	4	1.00	20	0.200
85	A	3	3	1.00	18	0.167
86	A	9	8	1.06	16	0.500
87	A	5	5	0.99	20	0.250
88	A	7	6	1.07	20	0.300
89	A	7	6	1.01	20	0.300
90	A	7	7	0.92	20	0.350

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
91	A	6	5	1.05	20	0.250
92	A	5	5	0.93	20	0.250
93	A	6	5	1.01	20	0.250
94	A	4	4	0.96	20	0.200
95	A	6	5	1.01	20	0.250
96	A	4	4	0.96	20	0.200
97	A	6	5	1.03	20	0.250
98	A	5	5	0.94	20	0.250
99	A	7	6	1.25	18	0.333
100	A	6	6	1.00	16	0.375
101	A	7	6	1.15	20	0.300
102	A	8	8	0.99	20	0.400
103	A	7	6	1.04	20	0.300
104	A	3	3	1.00	20	0.150
105	A	7	7	1.02	24	0.292
106	A	6	5	1.00	24	0.208
107	A	6	6	1.03	24	0.250
108	A	9	8	0.80	24	0.333
109	A	11	10	1.07	24	0.417
110	A	9	9	1.00	24	0.375
111	A	7	6	1.03	24	0.250
112	A	8	8	1.02	24	0.333
113	A	5	4	1.00	24	0.167
114	A	2	2	1.00	24	0.083
115	A	3	2	1.00	24	0.083
116	A	7	7	1.04	24	0.292
117	A	7	7	1.01	24	0.292
118	A	4	3	1.00	24	0.125
119	A	10	10	0.96	24	0.417
120	A	7	6	1.08	24	0.250
121	A	1	1	1.00	34	0.029
122	A	3	3	1.14	27	0.111
123	A	3	2	1.00	18	0.111
124	A	4	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	4	3	1.00	17	0.176
126	A	6	5	0.97	18	0.278
127	A	7	6	0.97	18	0.333
128	A	7	6	0.97	17	0.353
129	A	3	2	1.00	18	0.111
130	A	4	3	1.00	18	0.167
131	A	5	4	1.00	22	0.182
132	A	5	4	1.00	24	0.167
133	A	4	3	1.00	20	0.150
134	A	4	3	1.00	20	0.150
135	A	5	4	1.00	24	0.167
136	A	5	4	1.00	26	0.154
137	A	4	3	1.00	18	0.167
138	A	4	3	1.00	18	0.167
139	A	5	4	1.00	20	0.200
140	A	3	2	1.00	36	0.056

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(ax^2 + bx^3 + cx^4)\,dx$

3.1.1 Optimal result

Integrand size = 20, antiderivative size = 25

$$\int x^2\left(ax^2 + bx^3 + cx^4\right)\,dx = \frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

output
```
1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2 + bx^3 + cx^4\right)\,dx = \frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

3.1.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3+cx^4\right) \, dx$$
$$\Big\downarrow \text{9}$$
$$\int x^4 \left(a+bx+cx^2\right)dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(ax^4+bx^5+cx^6\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

input
```
Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

3.1.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^5(30c\,x^2+35bx+42a)}{210}$	20
default	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
norman	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
risch	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
parallelrisch	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20

input
```
int(x^2*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/210*x^5*(30*c*x^2+35*b*x+42*a)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\,dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

input
```
integrate(x**2*(c*x**4+b*x**3+a*x**2),x)
```

output
```
a*x**5/5 + b*x**6/6 + c*x**7/7
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\,dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\,dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\,dx=\frac{x^5\,(30\,c\,x^2+35\,b\,x+42\,a)}{210}$$

input
```
int(x^2*(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(x^5*(42*a + 35*b*x + 30*c*x^2))/210
```

3.2 $\int x(ax^2 + bx^3 + cx^4)\,dx$

3.2.1 Optimal result

Integrand size = 18, antiderivative size = 25

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

output
```
1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

input
```
Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

3.2.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx$$

$$\Big\downarrow \ 9$$

$$\int x^3\left(a+bx+cx^2\right)dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(ax^3+bx^4+cx^5\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

input
```
Int[x*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

3.2.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^4(10c\,x^2+12bx+15a)}{60}$	20
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
parallelrisch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20

input
```
int(x*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^4*(10*c*x^2+12*b*x+15*a)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3+cx^4)\,dx=\frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2+bx^3+cx^4)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2),x)
```

output
```
a*x**4/4 + b*x**5/5 + c*x**6/6
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3 + cx^4) \, dx = \frac{1}{6} cx^6 + \frac{1}{5} bx^5 + \frac{1}{4} ax^4$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3 + cx^4) \, dx = \frac{1}{6} cx^6 + \frac{1}{5} bx^5 + \frac{1}{4} ax^4$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3 + cx^4) \, dx = \frac{x^4 \left(10\, c\, x^2 + 12\, b\, x + 15\, a\right)}{60}$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(x^4*(15*a + 12*b*x + 10*c*x^2))/60
```

3.3 $\int (ax^2 + bx^3 + cx^4) \, dx$

3.3.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int (ax^2 + bx^3 + cx^4) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

output
```
1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ax^2 + bx^3 + cx^4) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

input
```
Integrate[a*x^2 + b*x^3 + c*x^4,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

3.3.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3+cx^4)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

input
```
Int[a*x^2 + b*x^3 + c*x^4,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^{3}\left(12 c \,x^{2}+15 b x +20 a \right)}{60}$	20
default	$\frac{1}{3} a \,x^{3}+\frac{1}{4} b \,x^{4}+\frac{1}{5} c \,x^{5}$	20
norman	$\frac{1}{3} a \,x^{3}+\frac{1}{4} b \,x^{4}+\frac{1}{5} c \,x^{5}$	20
risch	$\frac{1}{3} a \,x^{3}+\frac{1}{4} b \,x^{4}+\frac{1}{5} c \,x^{5}$	20
parallelrisch	$\frac{1}{3} a \,x^{3}+\frac{1}{4} b \,x^{4}+\frac{1}{5} c \,x^{5}$	20
parts	$\frac{1}{3} a \,x^{3}+\frac{1}{4} b \,x^{4}+\frac{1}{5} c \,x^{5}$	20

input
```
int(c*x^4+b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(12*c*x^2+15*b*x+20*a)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right) dx = \frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right) dx = \frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

input
```
integrate(c*x**4+b*x**3+a*x**2,x)
```

output
```
a*x**3/3 + b*x**4/4 + c*x**5/5
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right) dx = \frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3 + cx^4) \, dx = \frac{1}{5} cx^5 + \frac{1}{4} bx^4 + \frac{1}{3} ax^3$$

input
```
integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3 + cx^4) \, dx = \frac{x^3 (12 \, c \, x^2 + 15 \, b \, x + 20 \, a)}{60}$$

input
```
int(a*x^2 + b*x^3 + c*x^4,x)
```

output
```
(x^3*(20*a + 15*b*x + 12*c*x^2))/60
```

3.4 $\int \frac{ax^2+bx^3+cx^4}{x}\,dx$

3.4.1 Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

output
```
1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

3.4.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx$$
$$\Big\downarrow \; 9$$
$$\int x\left(a+bx+cx^2\right)dx$$
$$\Big\downarrow \; 1140$$
$$\int \left(ax+bx^2+cx^3\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

3.4.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^2(3c\,x^2+4bx+6a)}{12}$	20
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3+\frac{1}{4}c\,x^4$	20

input
```
int((c*x^4+b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*c*x^2+4*b*x+6*a)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx = \frac{1}{4}\,cx^4 + \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx = \frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x,x)
```

output
```
a*x**2/2 + b*x**3/3 + c*x**4/4
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x}\, dx = \frac{1}{4}\, cx^4 + \frac{1}{3}\, bx^3 + \frac{1}{2}\, ax^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x}\, dx = \frac{1}{4}\, cx^4 + \frac{1}{3}\, bx^3 + \frac{1}{2}\, ax^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x}\, dx = \frac{x^2\,(3\,c\,x^2 + 4\,b\,x + 6\,a)}{12}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x,x)
```

output
```
(x^2*(6*a + 4*b*x + 3*c*x^2))/12
```

3.5 $\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx$

3.5.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

3.5.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {9, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \left(a+bx+cx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

3.5.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$ax + \frac{1}{2}b\,x^2 + \frac{1}{3}c\,x^3$	17
risch	$ax + \frac{1}{2}b\,x^2 + \frac{1}{3}c\,x^3$	17
parallelrisch	$ax + \frac{1}{2}b\,x^2 + \frac{1}{3}c\,x^3$	17
parts	$ax + \frac{1}{2}b\,x^2 + \frac{1}{3}c\,x^3$	17
gosper	$\frac{x(2c\,x^2+3bx+6a)}{6}$	18
norman	$\frac{a\,x^2+\frac{1}{2}b\,x^3+\frac{1}{3}c\,x^4}{x}$	23

input
```
int((c*x^4+b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x^2+1/3*c*x^3
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2}\,dx = \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2}\,dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x**2,x)
```

output
```
a*x + b*x**2/2 + c*x**3/3
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx=\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x^2,x)
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

3.6 $\int x^2(ax^2+bx^3+cx^4)^2\,dx$

3.6.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

output
```
1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^
11)/11
```

3.6.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^6 \left(a+bx+cx^2\right)^2 dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(a^2x^6+x^8\left(2ac+b^2\right)+2abx^7+2bcx^9+c^2x^{10}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

input
```
Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^
11)/11
```

3.6.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^7}{7}+\frac{ab\,x^8}{4}+\frac{(2ac+b^2)x^9}{9}+\frac{bc\,x^{10}}{5}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{bc\,x^{10}}{5}+\left(\frac{2ac}{9}+\frac{b^2}{9}\right)x^9+\frac{ab\,x^8}{4}+\frac{a^2x^7}{7}$	46
gosper	$\frac{x^7(1260c^2x^4+2772bc\,x^3+3080ac\,x^2+1540b^2x^2+3465abx+1980a^2)}{13860}$	47
risch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{1}{5}bc\,x^{10}+\frac{1}{11}c^2x^{11}$	47
parallelrisch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{1}{5}bc\,x^{10}+\frac{1}{11}c^2x^{11}$	47

input
```
int(x^2*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{4}\,abx^8+\frac{1}{9}\,(b^2+2\,ac)x^9+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a
^2*x^7
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{bcx^{10}}{5}+\frac{c^2x^{11}}{11}+x^9\cdot\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

input
```
integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b
**2/9)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{4}\,abx^8+\frac{1}{9}\,(b^2+2\,ac)x^9+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a
^2*x^7
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{9}\,b^2x^9+\frac{2}{9}\,acx^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1
/7*a^2*x^7
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x^2 \left(ax^2+bx^3+cx^4\right)^2 \, dx = x^9 \left(\frac{b^2}{9}+\frac{2\,a\,c}{9}\right)+\frac{a^2\,x^7}{7}+\frac{c^2\,x^{11}}{11}+\frac{a\,b\,x^8}{4}+\frac{b\,c\,x^{10}}{5}$$

input
```
int(x^2*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x^9*((2*a*c)/9 + b^2/9) + (a^2*x^7)/7 + (c^2*x^11)/11 + (a*b*x^8)/4 + (b*c
*x^10)/5
```

3.7 $\int x(ax^2+bx^3+cx^4)^2\,dx$

3.7.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}(b^2+2ac)\,x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

output
```
1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}(b^2+2ac)\,x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

input
```
Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2
*x^10)/10
```

3.7.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^5\left(a+bx+cx^2\right)^2dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^2x^5+x^7\left(2ac+b^2\right)+2abx^6+2bcx^8+c^2x^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

input
```
Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2
*x^10)/10
```

3.7.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^6}{6}+\frac{2ab\,x^7}{7}+\frac{(2ac+b^2)x^8}{8}+\frac{2bc\,x^9}{9}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{2bc\,x^9}{9}+\left(\frac{ac}{4}+\frac{b^2}{8}\right)x^8+\frac{2ab\,x^7}{7}+\frac{a^2x^6}{6}$	46
gosper	$\frac{x^6(252c^2x^4+560bc\,x^3+630ac\,x^2+315b^2x^2+720abx+420a^2)}{2520}$	47
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{2}{9}bc\,x^9+\frac{1}{10}c^2x^{10}$	47
parallelrisch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{2}{9}bc\,x^9+\frac{1}{10}c^2x^{10}$	47

input
```
int(x*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{2}{7}\,abx^7+\frac{1}{8}\,(b^2+2\,ac)x^8+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^
2*x^6
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{2bcx^9}{9}+\frac{c^2x^{10}}{10}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 +
b**2/8)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{10}c^2x^{10}+\frac{2}{9}bcx^9+\frac{2}{7}abx^7+\frac{1}{8}(b^2+2ac)x^8+\frac{1}{6}a^2x^6$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^
2*x^6
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{10}c^2x^{10}+\frac{2}{9}bcx^9+\frac{1}{8}b^2x^8+\frac{1}{4}acx^8+\frac{2}{7}abx^7+\frac{1}{6}a^2x^6$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/
6*a^2*x^6
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx=x^8\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{a^2\,x^6}{6}+\frac{c^2\,x^{10}}{10}+\frac{2\,a\,b\,x^7}{7}+\frac{2\,b\,c\,x^9}{9}$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x^8*((a*c)/4 + b^2/8) + (a^2*x^6)/6 + (c^2*x^10)/10 + (2*a*b*x^7)/7 + (2*b
*c*x^9)/9
```

3.8 $\int (ax^2+bx^3+cx^4)^2\,dx$

3.8.1 Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

output
```
1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9
)/9
```

3.8.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1949, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2 + bx^3 + cx^4)^2 \, dx$$

$$\Big\downarrow \text{ 1949}$$

$$\int x^4 (a + bx + cx^2)^2 \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int (a^2x^4 + x^6(2ac + b^2) + 2abx^5 + 2bcx^7 + c^2x^8) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9
)/9
```

3.8.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.8.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^{2} x^{5}}{5}+\frac{a b \,x^{6}}{3}+\frac{\left(2 a c +b^{2}\right) x^{7}}{7}+\frac{b c \,x^{8}}{4}+\frac{c^{2} x^{9}}{9}$	45
norman	$\frac{c^{2} x^{9}}{9}+\frac{b c \,x^{8}}{4}+\left(\frac{2 a c}{7}+\frac{b^{2}}{7}\right) x^{7}+\frac{a b \,x^{6}}{3}+\frac{a^{2} x^{5}}{5}$	46
gosper	$\frac{x^{5} \left(140 c^{2} x^{4}+315 b c \,x^{3}+360 a c \,x^{2}+180 b^{2} x^{2}+420 a b x +252 a^{2}\right)}{1260}$	47
risch	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b \,x^{6}+\frac{2}{7} x^{7} a c +\frac{1}{7} b^{2} x^{7}+\frac{1}{4} b c \,x^{8}+\frac{1}{9} c^{2} x^{9}$	47
parallelrisch	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b \,x^{6}+\frac{2}{7} x^{7} a c +\frac{1}{7} b^{2} x^{7}+\frac{1}{4} b c \,x^{8}+\frac{1}{9} c^{2} x^{9}$	47

input
```
int((c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{1}{9}\,c^{2}x^{9}+\frac{1}{4}\,bcx^{8}+\frac{1}{3}\,abx^{6}+\frac{1}{7}\,(b^{2}+2\,ac)x^{7}+\frac{1}{5}\,a^{2}x^{5}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/3*a*b*x^6 + 1/7*(b^2 + 2*a*c)*x^7 + 1/5*a^2*
x^5
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^{2}x^{5}}{5}+\frac{abx^{6}}{3}+\frac{bcx^{8}}{4}+\frac{c^{2}x^{9}}{9}+x^{7}\cdot\left(\frac{2ac}{7}+\frac{b^{2}}{7}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c/7 + b**2
/7)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \left(ax^2+bx^3+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{7}\,b^2x^7+\frac{1}{5}\,a^2x^5+\frac{1}{21}\,\left(6\,cx^7+7\,bx^6\right)a$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c*x^7 + 7*
b*x^6)*a
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(ax^2+bx^3+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/5*
a^2*x^5
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \left(ax^2+bx^3+cx^4\right)^2 \, dx = x^7\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^5}{5}+\frac{c^2\,x^9}{9}+\frac{a\,b\,x^6}{3}+\frac{b\,c\,x^8}{4}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x^7*((2*a*c)/7 + b^2/7) + (a^2*x^5)/5 + (c^2*x^9)/9 + (a*b*x^6)/3 + (b*c*x
^8)/4
```

3.9 $\int \frac{(ax^2+bx^3+cx^4)^2}{x} dx$

3.9.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x} dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{6}\left(b^2+2ac\right)x^6 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x} dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{6}\left(b^2+2ac\right)x^6 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2
*x^8)/8
```

3.9.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x} dx$$

$$\Big\downarrow 9$$

$$\int x^3 (a+bx+cx^2)^2 dx$$

$$\Big\downarrow 1140$$

$$\int \left(a^2x^3+x^5(2ac+b^2)+2abx^4+2bcx^6+c^2x^7\right) dx$$

$$\Big\downarrow 2009$$

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6(2ac+b^2)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2
*x^8)/8
```

3.9.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^4}{4}+\frac{2ab\,x^5}{5}+\frac{(2ac+b^2)x^6}{6}+\frac{2bc\,x^7}{7}+\frac{c^2x^8}{8}$	45
norman	$\frac{c^2x^8}{8}+\frac{2bc\,x^7}{7}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{2ab\,x^5}{5}+\frac{a^2x^4}{4}$	46
gosper	$\frac{x^4(105c^2x^4+240bc\,x^3+280ac\,x^2+140b^2x^2+336abx+210a^2)}{840}$	47
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{2}{7}bc\,x^7+\frac{1}{8}c^2x^8$	47
parallelrisch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{2}{7}bc\,x^7+\frac{1}{8}c^2x^8$	47

input
```
int((c*x^4+b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x}\,dx = \frac{1}{8}\,c^2x^8+\frac{2}{7}\,bcx^7+\frac{2}{5}\,abx^5+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

output
```
1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*
x^4
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} \, dx = \frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

output
```
a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b*
*2/6)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} \, dx = \frac{1}{8} c^2x^8 + \frac{2}{7} bcx^7 + \frac{2}{5} abx^5 + \frac{1}{6} (b^2 + 2\,ac)x^6 + \frac{1}{4} a^2x^4$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

output
```
1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*
x^4
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} \, dx = \frac{1}{8} c^2x^8 + \frac{2}{7} bcx^7 + \frac{1}{6} b^2x^6 + \frac{1}{3} acx^6 + \frac{2}{5} abx^5 + \frac{1}{4} a^2x^4$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

output
```
1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x^5 + 1/4*
a^2*x^4
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} \, dx = x^6 \left(\frac{b^2}{6} + \frac{a\,c}{3} \right) + \frac{a^2\,x^4}{4} + \frac{c^2\,x^8}{8} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^7}{7}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2/x,x)
```

output
```
x^6*((a*c)/3 + b^2/6) + (a^2*x^4)/4 + (c^2*x^8)/8 + (2*a*b*x^5)/5 + (2*b*c
*x^7)/7
```

3.10 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx$

3.10.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{1}{5}\left(b^2+2ac\right)x^5 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{1}{5}\left(b^2+2ac\right)x^5 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7
)/7
```

3.10.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2} dx$$

$$\Big\downarrow \; 9$$

$$\int x^2 (a+bx+cx^2)^2 dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(a^2x^2+x^4(2ac+b^2)+2abx^3+2bcx^5+c^2x^6\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5(2ac+b^2)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7
)/7
```

3.10.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^{2} x^{3}}{3}+\frac{a b\,x^{4}}{2}+\frac{\left(2 a c +b^{2}\right) x^{5}}{5}+\frac{b c\,x^{6}}{3}+\frac{c^{2} x^{7}}{7}$	45
gosper	$\frac{x^{3} \left(30 c^{2} x^{4}+70 b c\,x^{3}+84 a c\,x^{2}+42 b^{2} x^{2}+105 a b x +70 a^{2}\right)}{210}$	47
risch	$\frac{1}{3} a^{2} x^{3}+\frac{1}{2} a b\,x^{4}+\frac{2}{5} x^{5} a c +\frac{1}{5} b^{2} x^{5}+\frac{1}{3} b c\,x^{6}+\frac{1}{7} c^{2} x^{7}$	47
parallelrisch	$\frac{1}{3} a^{2} x^{3}+\frac{1}{2} a b\,x^{4}+\frac{2}{5} x^{5} a c +\frac{1}{5} b^{2} x^{5}+\frac{1}{3} b c\,x^{6}+\frac{1}{7} c^{2} x^{7}$	47
norman	$\frac{\left(\frac{2 a c}{5}+\frac{b^{2}}{5}\right) x^{6}+\frac{a^{2} x^{4}}{3}+\frac{c^{2} x^{8}}{7}+\frac{a b\,x^{5}}{2}+\frac{b c\,x^{7}}{3}}{x}$	50

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{2}\,abx^4 + \frac{1}{5}\,\left(b^2+2\,ac\right)x^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*
x^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5 \cdot \left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

output
```
a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2
/5)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{2}\,abx^4 + \frac{1}{5}\,(b^2+2\,ac)x^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*
x^3
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*
a^2*x^3
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2} \, dx = x^5 \left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^7}{7} + \frac{a\,b\,x^4}{2} + \frac{b\,c\,x^6}{3}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2/x^2,x)
```

output
```
x^5*((2*a*c)/5 + b^2/5) + (a^2*x^3)/3 + (c^2*x^7)/7 + (a*b*x^4)/2 + (b*c*x
^6)/3
```

3.11 $\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$

3.11.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}$$

output
```
-b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = \frac{cx(-2b+cx)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log(a+x(b+cx))}{2c^3}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3}{a+bx+cx^2}dx$$

$$\Big\downarrow \; 1143$$

$$\int \left(\frac{x(b^2-ac)+ab}{c^2(a+bx+cx^2)}-\frac{b}{c^2}+\frac{x}{c}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

input
```
Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
-((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*
c^3)
```

3.11.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{2}c\,x^{2}+bx}{c^{2}}+\frac{\frac{\left(-ac+b^{2}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^{2}\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c^{2}}$
risch	$\frac{x^{2}}{2c}-\frac{bx}{c^{2}}-\frac{2\ln\left(12a^{2}b\,c^{2}-7a\,b^{3}c+b^{5}-2\sqrt{-b^{2}(4ac-b^{2})(3ac-b^{2})^{2}}\,cx-\sqrt{-b^{2}(4ac-b^{2})(3ac-b^{2})^{2}}\,b\right)a^{2}}{c(4ac-b^{2})}+\frac{5\ln\left(12a^{2}b\,c^{2}-7a\,b^{3}c\right.}{}$

input
```
int(x^5/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2
*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = \left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+(b}{2\,(b^2c^3-4\,ac^4)}\right.$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x
^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*
x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.

Time = 0.48 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.28

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = -\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \frac{x^2}{2c}$$

input
```
integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

output
```
-b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-
4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**
3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b*
*2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*
(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2
*a**2*c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*
(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2
)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*
c - b**3)) + x**2/(2*c)
```

3.11.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4}\, dx = \frac{cx^2 - 2\,bx}{2\,c^2} + \frac{(b^2 - ac)\log(cx^2 + bx + a)}{2\,c^3} - \frac{(b^3 - 3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^3}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3
- 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4}\, dx = \frac{x^2}{2\,c} - \frac{\ln(c\,x^2 + b\,x + a)\,(4\,a^2\,c^2 - 5\,a\,b^2\,c + b^4)}{2\,(4\,a\,c^4 - b^2\,c^3)} - \frac{b\,x}{c^2} + \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,(3\,a\,c - b^2)}{c^3\,\sqrt{4\,a\,c - b^2}}$$

input
```
int(x^5/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c
^4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a
*c - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

3.11. $\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$

3.12 $\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$

3.12.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx=\frac{x}{c}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}$$

output
```
x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx=\frac{x}{c}+\frac{(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^
2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

3.12.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax^2+bx^3+cx^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2}{a+bx+cx^2}dx$$

$$\Big\downarrow \; 1143$$

$$\int \left(\frac{1}{c}-\frac{a+bx}{c\left(a+bx+cx^2\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

input
```
Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

3.12.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,cx-\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,b\right)ab}{c(4ac-b^{2})}+\frac{\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4ac-\right.}{}$

input
```
int(x^4/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int\frac{x^{4}}{ax^{2}+bx^{3}+cx^{4}}\,dx$$
$$=\left[-\frac{(b^{2}-2\,ac)\sqrt{b^{2}-4\,ac}\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right)-2\,(b^{2}c-4\,ac^{2})x+(b^{3}-4\,abc)\log\left(cx^{2}\right.}{2\,(b^{2}c^{2}-4\,ac^{3})}\right.$$
$$-\frac{2\,(b^{2}-2\,ac)\sqrt{-b^{2}+4\,ac}\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)-2\,(b^{2}c-4\,ac^{2})x+(b^{3}-4\,abc)\log\left(cx^{2}+bx+a\right.}{2\,(b^{2}c^{2}-4\,ac^{3})}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c
^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2
*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x +
a))/(b^2*c^2 - 4*a*c^3)]
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx = \left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right.}{2ac-b^2}\right. + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right.}{2ac-b^2}\right. + \frac{x}{c}$$

input
```
integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

output
```
(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))
*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4
*a*c**2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c -
 b**2))) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**
2*(4*a*c - b**2))))/(2*a*c - b**2)) + x/c
```

3.12.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4}\,dx = \frac{x}{c} - \frac{b\log(cx^2 + bx + a)}{2\,c^2} + \frac{(b^2 - 2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^2}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4}\,dx = \frac{x}{c} + \frac{b^3\,\ln(c\,x^2 + b\,x + a)}{2\,(4\,a\,c^3 - b^2\,c^2)} - \frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c - b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c - b^2}} - \frac{2\,a\,b\,c\,\ln(c\,x^2 + b\,x + a)}{4\,a\,c^3 - b^2\,c^2}$$

input
```
int(x^4/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*
a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) +
 (b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a
*c - b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

3.12. $\int \frac{x^4}{ax^2+bx^3+cx^4} \, dx$

3.13 $\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx$

3.13.1 Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

output
```
1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^
2)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log(a+x(b+cx))}{2c}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
+ x*(b + c*x)])/(2*c)
```

3.13.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax^2+bx^3+cx^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x}{a+bx+cx^2}dx$$

$$\Big\downarrow \; 1142$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{b\int \frac{1}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \; 1083$$

$$\frac{b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}+\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \; 219$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}+\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 1103$$

$$\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

input
```
Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
 b*x + c*x^2]/(2*c)
```

3.13.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.13.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result
default	$\frac{\ln(c\,x^2+bx+a)}{2c}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)a}{4ac-b^2}-\frac{\ln\left(-2\sqrt{-b^2(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)b^2}{2c(4ac-b^2)}+\frac{\ln\left(-2\cdots\right.}{}$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.30

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)}, \frac{2\,\sqrt{-b^2+4\,ac}b\,\mathrm{arcta}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b
^2*c - 4*a*c^2)]
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.

Time = 0.18 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.86

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)} + \frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-
b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqr
t(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b*
*2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*
(4*a*c - b**2)) + 1/(2*c)))/b)
```

3.13.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{ax^2 + bx^3 + cx^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{ax^2 + bx^3 + cx^4} \, dx = -\frac{b \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,acc}} + \frac{\log\left(cx^2 + bx + a\right)}{2\,c}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log
(c*x^2 + b*x + a)/c
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} - \frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(
1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a
+ b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

3.14 $\int \frac{x^2}{ax^2+bx^3+cx^4} dx$

3.14.1 Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x^2}{ax^2+bx^3+cx^4} dx = -\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{ax^2+bx^3+cx^4} dx = \frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

3.14.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax^2+bx^3+cx^4}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{a+bx+cx^2}dx$$
$$\Big\downarrow \text{ 1083}$$
$$-2\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

3.14.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.14.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

input
```
int(x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(34) = 68.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sq
rt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c
*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

3.14.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2
)^(1/2)
```

3.15 $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx+cx^2\right)}{2a}$$

output
```
ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-
4*a*c+b^2)^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx = -\frac{\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2\log(x) + \log(a+x(b+cx))}{2a}$$

input
```
Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*
Log[x] + Log[a + x*(b + c*x)])/a
```

3.15.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {9, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^2+bx^3+cx^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x\,(a+bx+cx^2)}dx$$

$$\Big\downarrow \text{ 1144}$$

$$\frac{\int -\frac{b+cx}{cx^2+bx+a}dx}{a}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\log(x)}{a}-\frac{\int \frac{b+cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^2+bx+a}dx+\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\log\left(a+bx+cx^2\right)-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

input
```
Int[x/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
Log[x]/a - (-((b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
) + Log[a + b*x + c*x^2]/2)/a
```

3.15.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.15.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$\frac{\ln(x)}{a}+\frac{-\frac{\ln\left(c\,x^2+bx+a\right)}{2}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a}$
risch	$-\frac{2\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-b^2(4ac-b^2)}\,ac-2\sqrt{-b^2(4ac-b^2)}\,b^2\right)x+12cb\,a^2-3a\,b^3-\sqrt{-b^2(4ac-b^2)}\,ab\right)c}{4ac-b^2}+\frac{\ln\left(\left(8a\,b^2c-2b^4+6\sqrt{-b\right.\right.}{}$

input
```
int(x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a+1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2)))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log\left(x\right)}{2\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*
c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)
*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.

Time = 4.49 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right) \cdots}{9abc^2 - 2 \cdots}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right) + 2a^2 \cdots}{9abc^2 - 2b^3c}\right) + \frac{\log(x)}{a}$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c
**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b
**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3
*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**
4*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2
*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2
+ 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/
(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b*
*2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*
b*c**2 - 2*b**3*c)) + log(x)/a
```

3.15.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx = -\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log
(c*x^2 + b*x + a)/a + log(abs(x))/a
```

3.15.9 Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx = \frac{\ln(x)}{a} - \ln\left(\left(b\,c - \left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}-\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right) + 3\,c^2\,x\right)\left(\frac{1}{2\,a}-\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)\right) - \ln\left(\left(\left(x\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\left(\frac{1}{2\,a}+\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right) - b\,c - 3\,c^2\,x\right)\left(\frac{1}{2\,a}+\frac{b\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)\right)$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 -
4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c
)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(
2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/
(2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

3.15. $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

3.16 $\int \frac{1}{ax^2+bx^3+cx^4}\,dx$

3.16.1 Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx=-\frac{1}{ax}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx=\frac{-\frac{2a}{x}+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2b\log(x)+b\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

output
```
((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

3.16.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1949, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx^3+cx^4}dx$$
$$\Big\downarrow \text{ 1949}$$
$$\int \frac{1}{x^2(a+bx+cx^2)}dx$$
$$\Big\downarrow \text{ 1145}$$
$$\frac{\int -\frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\int \frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$
$$\Big\downarrow \text{ 1200}$$
$$-\frac{\int \left(\frac{b}{ax}+\frac{-b^2-cxb+ac}{a(cx^2+bx+a)}\right)dx}{a}-\frac{1}{ax}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2a}+\frac{b\log(x)}{a}}{a}-\frac{1}{ax}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

output
```
-(1/(a*x)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sq
rt[b^2 - 4*a*c]) + (b*Log[x])/a - (b*Log[a + b*x + c*x^2])/(2*a))/a
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{\frac{b\ln\left(cx^2+bx+a\right)}{2}+\frac{2\left(-ac+\frac{b^2}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}$
risch	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(6a^3c-2a^2b^2\right)\_R^2-2\_Rabc\right.\right.\right.$

input
```
int(1/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x-b*ln(x)/a^2+1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

$$\int \frac{1}{ax^2 + bx^3 + cx^4}\, dx$$

$$= \left[-\frac{(b^2 - 2\,ac)\sqrt{b^2 - 4\,ac}x \log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac + \sqrt{b^2 - 4\,ac}(2\,cx + b)}{cx^2 + bx + a}\right) + 2\,ab^2 - 8\,a^2c - (b^3 - 4\,abc)x \log\left(cx^2 \right.}{2\,(a^2b^2 - 4\,a^3c)x} \right.$$

$$-\frac{2\,(b^2 - 2\,ac)\sqrt{-b^2 + 4\,ac}x \arctan\left(-\frac{\sqrt{-b^2 + 4\,ac}(2\,cx + b)}{b^2 - 4\,ac}\right) + 2\,ab^2 - 8\,a^2c - (b^3 - 4\,abc)x \log\left(cx^2 + bx + a\right.}{2\,(a^2b^2 - 4\,a^3c)x}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^
2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))
/((a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^
3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^
2 - 4*a^3*c)*x)]
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{ax^2 + bx^3 + cx^4}\, dx = \text{Timed out}$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2),x)
```

output
```
Timed out
```

3.16.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{ax^2 + bx^3 + cx^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{ax^2 + bx^3 + cx^4} \, dx = \frac{b \log (cx^2 + bx + a)}{2 a^2} - \frac{b \log (|x|)}{a^2} + \frac{(b^2 - 2 ac) \arctan \left(\frac{2 cx + b}{\sqrt{-b^2 + 4 ac}}\right)}{\sqrt{-b^2 + 4 ac} a^2} - \frac{1}{ax}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.19

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx$$
$$= \frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\right.}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{1}{a\,x}$$
$$-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\right.}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{b\,\ln(x)}{a^2}$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)
^(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c
*x + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b
^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(
2*a*b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2
) + 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x -
4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2))
+ (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

3.17 $\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx$

3.17.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3
+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/
2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx = \frac{-\frac{a^2}{x^2}+\frac{2ab}{x}-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x)+(-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

output
```
(-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a
+ x*(b + c*x)])/(2*a^3)
```

3.17.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^3\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \; 1145$$

$$\frac{\int -\frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \; 1200$$

$$-\frac{\int \left(\frac{b}{ax^2}+\frac{ac-b^2}{a^2x}+\frac{b(b^2-2ac)+c(b^2-ac)x}{a^2(cx^2+bx+a)}\right)dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^2}-\frac{\log(x)(b^2-ac)}{a^2}-\frac{b}{ax}}{a}-\frac{1}{2ax^2}$$

input
```
Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

output
```
-1/2*1/(a*x^2) - (-(b/(a*x)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x])/a^2 + ((b^2 -
 a*c)*Log[a + b*x + c*x^2])/(2*a^2))/a
```

3.17.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

method	result
default	$-\frac{1}{2a\,x^{2}}+\frac{\left(-ac+b^{2}\right)\ln \left(x \right)}{a^{3}}+\frac{b}{a^{2}x}+\frac{\frac{\left(a\,c^{2}-b^{2}c \right)\ln \left(c\,x^{2}+bx+a \right)}{2c}+\frac{2\left(2abc-b^{3}-\frac{\left(a\,c^{2}-b^{2}c \right)b}{2c}\right)\arctan \left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{3}}$
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}-\frac{\ln \left(x \right)c}{a^{2}}+\frac{\ln \left(x \right)b^{2}}{a^{3}}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^{4}-a^{3}b^{2}\right)\_Z^{2}+\left(-4a^{2}c^{2}+5a\,b^{2}c-b^{4}\right)\_Z+c^{3}\right)}\_R\ln \left(\left(\left(\left(6c\,a^{5}-2b^{2}a^{4}\right.\right.\right.\right.$

input
```
int(1/x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+(-a*c+b^2)*ln(x)/a^3+b/a^2/x+1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^
2+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx$$
$$= \Bigg[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+(b^4-5\,ab^2c+4\,a^2c^2)}{2\,(a^3b^2-4\,a^4c)x^2}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 -
4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4
- 5*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 -
 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*
c^2)*x^2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

3.17.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

output
```
Timed out
```

3.17.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx = -\frac{\left(b^2-ac\right)\log\left(cx^2+bx+a\right)}{2\,a^3} + \frac{\left(b^2-ac\right)\log\left(|x|\right)}{a^3} - \frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3} + \frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 -
(b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx$$

$$= \frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-}\right.}{4}$$

$$-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b^2}\right.}{}$$

$$-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln\left(x\right)\,\left(a\,c-b^2\right)}{a^3}$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)),x)
```

output
```
(log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x
*(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c
)^(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b
^2 - 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2
) + (b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2
*a*b^4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2
- 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2
) + 9*a^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4
*a*c)^(1/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b
^3*(b^2 - 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (
b*x)/a^2)/x^2 - (log(x)*(a*c - b^2))/a^3
```

3.18 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx$

3.18.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

output
```
-1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-
2*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx$$
$$=\frac{-\frac{2a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{6a(-b^2+ac)}{x}+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log(x)+3(b^3-2abc)\log(a+x}{6a^4}$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

output
```
((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] -
 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4
)
```

3.18.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax^2+bx^3+cx^4\right)} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^4 \left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \; 1145$$

$$\frac{\int -\frac{b+cx}{x^3 (cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{b+cx}{x^3 (cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \; 1200$$

$$-\frac{\int \left(\frac{b}{ax^3}+\frac{b^3-2abc}{a^3x}+\frac{-b^4+3acb^2-c\left(b^2-2ac\right)xb-a^2c^2}{a^3 (cx^2+bx+a)}+\frac{ac-b^2}{a^2x^2}\right)dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{b\log(x)\left(b^2-2ac\right)}{a^3}+\frac{b^2-ac}{a^2x}+\frac{\left(2a^2c^2-4ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{b}{2ax^2}}{a}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

output
```
-1/3*1/(a*x^3) - (-1/2*b/(a*x^2) + (b^2 - a*c)/(a^2*x) + ((b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*
c]) + (b*(b^2 - 2*a*c)*Log[x])/a^3 - (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2]
)/(2*a^3))/a
```

3.18.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.18.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$-\frac{1}{3a\,x^3}-\frac{-ac+b^2}{x\,a^3}+\frac{b(2ac-b^2)\ln(x)}{a^4}+\frac{b}{2a^2x^2}+\frac{\frac{\left(-2ab\,c^2+b^3c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(a^2c^2-3a\,b^2c+b^4-\frac{\left(-2ab\,c^2+b^3c\right)b}{2c}\right)\arctan\left(\right.}{\sqrt{4ac-b^2}}}{a^4}$
risch	Expression too large to display

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3-(-a*c+b^2)/x/a^3+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2+1/a^4*(1
/2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1/2*(-2*a
*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.25

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx$$
$$=\left[\frac{3\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\left(b^5-6\,a\right.}{}\right.$$
$$-\frac{6\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\left(b^5-6\,ab^3c+8\right.}{6\,(a}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2
 + b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*
a^2*b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*
c)*x^3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arct
an(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c -
3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*
b^3*c + 8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2
- 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

output
```
Timed out
```

3.18.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = \frac{(b^3-2\,abc)\log(cx^2+bx+a)}{2\,a^4} - \frac{(b^3-2\,abc)\log(|x|)}{a^4} + \frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^4} + \frac{3\,a^2bx-2\,a^3-6\,(ab^2-a^2c)x^2}{6\,a^4x^3}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))
/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^
2)/(a^4*x^3)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.82

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = \ln\Big(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}-17\,a^2\,b^2\,c^2\,x+12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Big)\left(\frac{b^3}{2\,a^4}-\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}+\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\ln\Big(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-11\,a^2\,b^3\,c+13\,a^3\,b\,c^2-2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}+17\,a^2\,b^2\,c^2\,x-12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\Big)\left(\frac{b^3}{2\,a^4}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}-\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\frac{\frac{x^2\,(a\,c-b^2)}{a^3}-\frac{1}{3\,a}+\frac{b\,x}{2\,a^2}}{x^3}+\frac{b\,\ln(x)\,(2\,a\,c-b^2)}{a^4}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x)
```

output
```
log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c
)^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c
)^(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2
) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(
b^3/(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^
2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(
b^2 - 4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b
*c^2 - 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a
*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/
2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^
(1/2))/(2*a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*
b^2)) + ((x^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*
(2*a*c - b^2))/a^4
```

3.18. $\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx$

3.19 $\int \frac{x^8}{(ax^2+bx^3+cx^4)^2} dx$

3.19.1 Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2} dx = \frac{2(b^2-3ac)x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} - \frac{b\log(a+bx+cx^2)}{c^3}$$

output
```
2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a
*c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

3.19.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2} dx = \frac{cx + \frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))} - \frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} - b\log(a+x(b+cx))}{c^3}$$

input
```
Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

3.19.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 1164, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^4}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \ 1164$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{2x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\int \frac{x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \ 1200$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\int \left(-\frac{b^2-3ac}{c^2}+\frac{bx}{c}+\frac{a(b^2-3ac)+b(b^2-4ac)x}{c^2(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \ 2009$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\left(\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2c^3}-\frac{x(b^2-3ac)}{c^2}+\frac{bx^2}{2c}\right)}{b^2-4ac}$$

input
```
Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(-(((b^2 - 3*a*c)
*x)/c^2) + (b*x^2)/(2*c) + ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c
*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)))/(b^2 - 4*a*c)
```

3.19.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.19.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{x}{c^2}-\frac{\frac{-\frac{(2a^2c^2-4a\,b^2c+b^4)x}{c(4ac-b^2)}+\frac{ba(3ac-b^2)}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{\frac{(4abc-b^3)\ln(c\,x^2+bx+a)}{c}+\frac{4\left(3c\,a^2-b^2a-\frac{(4abc-b^3)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$	198
risch	Expression too large to display	1176

input
```
int(x^8/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)
/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*
x+a)+2*(3*c*a^2-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(146) = 292.

Time = 0.28 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.58

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$$

$$= \left[-\frac{ab^5-7\,a^2b^3c+12\,a^3bc^2-(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^3-(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2+(ab^4-6\,a^2b^2c}{}\right.$$

$$-\frac{ab^5-7\,a^2b^3c+12\,a^3bc^2-(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^3-(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2+2\,(ab^4-6\,a^2b^2}{}$$

input
```
integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c +
 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*
a^2*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a
^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)
, -(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c +
 6*a^2*b*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2
 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 +
(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(141) = 282$.

Time = 0.99 (sec) , antiderivative size = 842, normalized size of antiderivative = 5.61

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \left(-\frac{b}{c^3} - \frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}\right.}{}\right.$$
$$+\left(-\frac{b}{c^3} + \frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}+\frac{\sqrt{-(4ac-b^2)^3}\cdot(6a^2c^2-6ab^2c+b^4)}{c^3\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}\right.}{}\right.$$
$$+\frac{-3a^2bc+ab^3+x(2a^2c^2-4ab^2c+b^4)}{4a^2c^4-ab^2c^3+x^2\cdot(4ac^5-b^2c^4)+x\,(4abc^4-b^3c^3)}+\frac{x}{c^2}$$

input
```
integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**
3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a
**2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a
**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16
*a**2*c**4*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
 b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2
*a*b**3 + 8*a*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) - b**4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6
*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 +
x*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*
a*c**5 - b**2*c**4) + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

3.19.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3
 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b
*x + a)*(b^2 - 4*a*c)*c^2)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.74

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{x}{c^2}+\frac{\frac{a\,(b^3-3\,a\,b\,c)}{c\,(4\,a\,c-b^2)}+\frac{x\,(2\,a^2\,c^2-4\,a\,b^2\,c+b^4)}{c\,(4\,a\,c-b^2)}}{c^3\,x^2+b\,c^2\,x+a\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)}-\frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{(4\,a\,c-b^2)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^8/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a
*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c
*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^
6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b
^2)^(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

3.20 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^2} dx$

3.20.1 Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2} dx = -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}} + \frac{\log(a+bx+cx^2)}{2c^2}$$

output
```
-b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^
2+b*x+a)/c^2
```

3.20.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2} dx = \frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))
) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

3.20.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^3}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{x(4a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \left(\frac{b}{c}-\frac{ab+(b^2-4ac)x}{c(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2c^2}+\frac{bx}{c}}{b^2-4ac}$$

input
```
Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*x)/c - (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) -
((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*c^2))/(b^2 - 4*a*c)
```

3.20.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{b\left(3ac-b^{2}\right)x}{c^{2}\left(4ac-b^{2}\right)}+\frac{a\left(2ac-b^{2}\right)}{\left(4ac-b^{2}\right)c^{2}}}{c\,x^{2}+bx+a}+\frac{\frac{\left(4ac-b^{2}\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-ab-\frac{\left(4ac-b^{2}\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c(4ac-b^{2})}$
risch	$\frac{\frac{b\left(3ac-b^{2}\right)x}{c^{2}\left(4ac-b^{2}\right)}+\frac{a\left(2ac-b^{2}\right)}{\left(4ac-b^{2}\right)c^{2}}}{c\,x^{2}+bx+a}+\frac{8\ln\left(-24a^{2}b\,c^{2}+10a\,b^{3}c-b^{5}-2\sqrt{-b^{2}(4ac-b^{2})(6ac-b^{2})^{2}}\,cx-\sqrt{-b^{2}(4ac-b^{2})(6ac-b^{2})^{2}}\,b\right)a^{2}}{(4ac-b^{2})^{2}}-\frac{4\ln}{}$

input
```
int(x^7/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(b/c^2*(3*a*c-b^2)/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x
+a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*c-
b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 635, normalized size of antiderivative = 5.57

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + (ab^3 - 6\,a^2bc + (b^3c - 6\,abc^2)x^2 + (b^4 - 6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx}{}\right.}{2\,(ab^4c^2 - 8\,a^2b^2c^3}\right.$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^
4)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (
b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2
*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

Time = 0.73 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.39

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{2c^2}\right)\log\left(x + \frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{2c^2}\right) + 8a^2c + 8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12}\right.}{6abc-b^3}\right.$$

$$+\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{2c^2}\right)\log\left(x + \frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{2c^2}\right) + 8a^2c + 8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-}\right.}{6abc-b^3}\right.$$

$$+\frac{2a^2c-ab^2+x(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2\cdot(4ac^4-b^2c^3)+x\,(4abc^3-b^3c^2)}$$

input
```
integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**
2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*
a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) +
x*(4*a*b*c**3 - b**3*c**2))
```

3.20.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^
3)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c +
 (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{a\,(2\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}+\frac{b\,x\,(3\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^7/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
 b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2
*b^2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4
*a*c - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^
4)))/(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

3.21 $\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx$

3.21.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*a*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(3/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{b^2x+a(b-2cx)}{c(-b^2+4ac)(a+x(b+cx))} + \frac{4a\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.21.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1153$$

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2a\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \; 1083$$

$$\frac{4a\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

3.21.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2a\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2a\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int(x^6/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-
b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.78

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[-\frac{ab^3-4\,a^2bc+2\,(ac^2x^2+abcx+a^2c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^4-6\,ab^2c}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}\right.$$
$$-\frac{ab^3-4\,a^2bc-4\,(ac^2x^2+abcx+a^2c)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^
2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x +
a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3
*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*x)]
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.18

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) +2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) +\frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

input
```
integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
-2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*
a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x +
 (32*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c
- b**2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b
 + x*(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2
) + x*(4*a*b*c**2 - b**3*c))
```

3.21.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.01

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,a\,c\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,a}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(x^6/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
- ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
 + c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c
*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

3.22 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$

3.22.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)} - \frac{2b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2a+bx}{(b^2-4ac)(a+x(b+cx))} - \frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.22.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1159}$$

$$\frac{b\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}+\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

input
```
Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

3.22.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

3.22.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70
risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left(\left(-8a\,c^2+2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(8a\,c^2-2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	148

input
```
int(x^5/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.12

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[\frac{2\,ab^2-8\,a^2c-(bcx^2+b^2x+ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^3-4\,abc)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}, \; 2\,ab\right.$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b
^2 - 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*x)]
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(60) = 120.

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.83

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$-b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$+\frac{-2a-bx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x\,(4abc-b^3)}$$

input
```
integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**
2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*
c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) +
b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c
- a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

3.22.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.22.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{2a}{4ac-b^2}+\frac{bx}{4ac-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{(4\,a\,c-b^2)^{3/2}}+\frac{2\,b\,c\,x}{(4\,a\,c-b^2)^{3/2}}\right)(4\,a\,c-b^2)}{b}\right)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int(x^5/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
- ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*ata
n(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2)
)/b))/(4*a*c - b^2)^(3/2)
```

3.23 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx$

3.23.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx=-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{4c\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx=-\frac{\frac{b+2cx}{a+x(b+cx)}+\frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
-(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

3.23.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1086}$$

$$-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{4c\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}$$

input
```
Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

3.23.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2cx+b}{(4ac-b^2)(c\,x^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	68
risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{c\,x^2+bx+a}+\frac{2c\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8a\,c^2-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	14

input
```
int(x^4/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b
^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 -
 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(61) = 122.

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx =$$
$$-2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)$$
$$+2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)$$
$$+\frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x\,(4abc-b^3)}$$

input
```
integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
-2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*
a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x
+ (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b
 + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**
3))
```

3.23.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{(ax^2 + bx^3 + cx^4)^2} \, dx = -\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2 - 4\,ac)\sqrt{-b^2 + 4\,ac}} - \frac{2\,cx + b}{(cx^2 + bx + a)(b^2 - 4\,ac)}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c^2\,x}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int(x^4/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
(b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan(((
(2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))
*(4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

3.24 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx$

3.24.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx=\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

output
```
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*
x+a)/a^2
```

3.24.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx=\frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2\log(x)-\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

3.24.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow 9$$

$$\int \frac{1}{x(a+bx+cx^2)^2}dx$$

$$\Big\downarrow 1165$$

$$\frac{-2ac+b^2+bcx}{a(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{a(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow 1200$$

$$\frac{\int \left(\frac{b^2-4ac}{ax}+\frac{-b(b^2-5ac)-c(b^2-4ac)x}{a(cx^2+bx+a)}\right)dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{a(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow 2009$$

$$\frac{\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2a}+\frac{\log(x)(b^2-4ac)}{a}}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{a(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((b*(b^2 - 6*a
*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2
- 4*a*c)*Log[x])/a - ((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a))/(a*(b^2 -
 4*a*c))
```

3.24.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

method	result
default	$\frac{\ln(x)}{a^2}-\frac{\frac{\frac{abcx}{4ac-b^2}-\frac{a(2ac-b^2)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(4a\,c^2-b^2c)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(5abc-b^3-\frac{(4a\,c^2-b^2c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}$
risch	$\frac{-\frac{bxc}{a(4ac-b^2)}+\frac{2ac-b^2}{a(4ac-b^2)}}{c\,x^2+bx+a}+\frac{\ln(x)}{a^2}+\left(\sum_{\_R=\mathrm{RootOf}((64a^5c^3-48a^4b^2c^2+12a^3b^4c-a^2b^6)\_Z^2+(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6)\_\right.$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2-1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*
x+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1
/2*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))))
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(102) = 204.

Time = 0.33 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.23

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$=\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx}{}\right.}{}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x
^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*
c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6
*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*x)*log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*l
og(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 +
16*a^4*c^3)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

3.24.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*
c)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 +
(a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.74

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{\frac{2\,a\,c-b^2}{a\,(4\,a\,c-b^2)} - \frac{b\,c\,x}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}$$
$$+ \frac{\ln\left(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}+84\,a^3\,b^2\,c^2\cdots\right.}{\cdots}$$
$$+ \frac{\ln\left(96\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}+23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}-84\,a^3\,b^2\,c^2\cdots\right.}{\cdots}$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output

```
log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))
/(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a
^3*b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24
*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b
^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3
)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))
/(2*a^2*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(
-(4*a*c - b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2)
- 84*a^3*b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
) + 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x -
12*a*b^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(
1/2)))/(2*a^2*(4*a*c - b^2)^3)
```

3.24. $\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx$

3.25 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx$

3.25.1 Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx = -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+cx^2)}{a^3}$$

output
```
-2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x
^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

3.25.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx = -\frac{\frac{a}{x} + \frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2b\log(x) - b\log(a+x(b+cx))}{a^3}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
-((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^
3)
```

3.25.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^2(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1165$$

$$\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{2(b^2+cxb-3ac)}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{b^2+cxb-3ac}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 1200$$

$$\frac{2\int \left(\frac{b^2-3ac}{ax^2}+\frac{4abc-b^3}{a^2x}+\frac{b^4-5acb^2+c(b^2-4ac)xb+3a^2c^2}{a^2(cx^2+bx+a)}\right)dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2a^2}-\frac{b\log(x)(b^2-4ac)}{a^2}-\frac{b^2-3ac}{ax}\right)}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) + (2*(-((b^2 -
 3*a*c)/(a*x)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[x])/a^2 + (b*
(b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c))
```

3.25.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.25.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

method	result
default	$-\frac{1}{a^2 x}-\frac{2b\ln(x)}{a^3}-\frac{\frac{\frac{ac(2ac-b^2)x}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{c x^2+bx+a}+\frac{\frac{(-4ab c^2+b^3c)\ln(c x^2+bx+a)}{c}+\frac{4\left(3a^2c^2-5a b^2c+b^4-\frac{(-4ab c^2+b^3c)b}{2c}\right)\arctan\left(\frac{2c}{\sqrt{4c}}\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3}$
risch	$\frac{-\frac{2c(3ac-b^2)x^2}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x}{a^2(4ac-b^2)}-\frac{1}{a}}{x(c x^2+bx+a)}-\frac{2b\ln(x)}{a^3}+2\left(\sum_{\_R=\mathrm{RootOf}((64a^6c^3-48a^5b^2c^2+12a^4b^4c-a^3b^6)\_Z^2+(-64b c^3a^3+48b^3c^2a}\right.$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-2*b*ln(x)/a^3-1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^
2)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c
*x^2+b*x+a)+2*(3*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(144) = 288$.

Time = 0.42 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.59

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx$$

$$= \left[-\frac{a^2b^4-8a^3b^2c+16a^4c^2+2(ab^4c-7a^2b^2c^2+12a^3c^3)x^2+((b^4c-6ab^2c^2+6a^2c^3)x^3+(b^5-6ab^3c}{}\right.$$

$$-\frac{a^2b^4-8a^3b^2c+16a^4c^2+2(ab^4c-7a^2b^2c^2+12a^3c^3)x^2+2((b^4c-6ab^2c^2+6a^2c^3)x^3+(b^5-6ab^3c}{}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6
*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6
*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c
 + 6*a^3*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^...
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(ax^2 + bx^3 + cx^4\right)^2}\, dx = \text{Timed out}$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

3.25.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2} \, dx = & \frac{2\,(b^4 - 6\,ab^2c + 6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2 - 4\,a^4c)\sqrt{-b^2 + 4\,ac}} \\ & - \frac{2\,b^2cx^2 - 6\,ac^2x^2 + 2\,b^3x - 7\,abcx + ab^2 - 4\,a^2c}{(a^2b^2 - 4\,a^3c)(cx^3 + bx^2 + ax)} \\ & + \frac{b\log\left(cx^2 + bx + a\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3}\end{aligned}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a
^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3
*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*
x)) + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

3.25.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 775, normalized size of antiderivative = 5.24

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \ln\Bigg(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\Bigg)\,\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}+\frac{b}{a^3}\right)-\frac{\frac{1}{a}-\frac{x\,\left(2\,b^3-7\,a\,b\,c\right)}{a^2\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,x^2\,\left(3\,a\,c-b^2\right)}{a^2\,\left(4\,a\,c-b^2\right)}}{c\,x^3+b\,x^2+a\,x}$$
$$-\ln\Bigg(2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-2\,b^8\,x-2\,a\,b^7+23\,a^2\,b^5\,c+108\,a^4\,b\,c^3-24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-97\,a^2\,b^4\,c^2\,x+138\,a^3\,b^2\,c^3\,x+24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\Bigg)\,\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}-\frac{b}{a^3}\right)-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c -
108*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b
^3*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*
c*x*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((
b^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (
2*c*x^2*(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*
a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^
4*b*c^3 - 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2
 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/
2) - 97*a^2*b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-
(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-
(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2) - b/a^3) - (2*b*log(x))/a^3
```

3.25. $\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx$

3.26 $\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 202

$$\begin{aligned}\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^2}\,dx =& -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2}+\frac{b(3b^2-11ac)}{a^3\left(b^2-4ac\right)x} \\ &+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} \\ &+\frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}} \\ &+\frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}\end{aligned}$$

output
```
1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/
x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^
2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-
2*a*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

3.26.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \frac{-\frac{a^2}{x^2}+\frac{4ab}{x}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2(3b^2-2ac)\log(x)+(-}{2a^4}$$

input
```
Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*
a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c +
 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
+ 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4
)
```

3.26.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{1}{x^3(a+bx+cx^2)^2}dx$$
$$\Big\downarrow \ 1165$$
$$\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}$$
$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 1200$$

$$\frac{\int \left(\frac{3b^2-8ac}{ax^3}+\frac{(b^2-4ac)(3b^2-2ac)}{a^3x}+\frac{-b(3b^4-17acb^2+19a^2c^2)-c(3b^4-14acb^2+8a^2c^2)x}{a^3(cx^2+bx+a)}+\frac{b(11ac-3b^2)}{a^2x^2}\right)dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-4ac)(3b^2-2ac)}{a^3}+\frac{b(3b^2-11ac)}{a^2x}+\frac{b(30a^2c^2-20ab^2c+3b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{3b^2}{2}\cdots}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)) + (-1/2*(3*b
^2 - 8*a*c)/(a*x^2) + (b*(3*b^2 - 11*a*c))/(a^2*x) + (b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*
a*c]) + ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[x])/a^3 - ((b^2 - 4*a*c)*(3*b^2
 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^3))/(a*(b^2 - 4*a*c))
```

3.26.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
 + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
 + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

method	result
default	$-\frac{1}{2a^2x^2}+\frac{(-2ac+3b^2)\ln(x)}{a^4}+\frac{2b}{a^3x}+\frac{\frac{\frac{acb(3ac-b^2)x}{4ac-b^2}-\frac{a(2a^2c^2-4a\,b^2c+b^4)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(8a^2c^3-14b^2a\,c^2+3b^4c)\ln(c\,x^2+bx+a)}{2c}+\frac{2(19a^2b\,c^2 \cdots}{\cdots}}{\cdots}}{a^4}$
risch	$\frac{\frac{bc(11ac-3b^2)x^3}{a^3(4ac-b^2)}-\frac{(8a^2c^2-25a\,b^2c+6b^4)x^2}{2a^3(4ac-b^2)}+\frac{3bx}{2a^2}-\frac{1}{2a}}{(c\,x^2+bx+a)x^2}-\frac{2\ln(x)c}{a^3}+\frac{3b^2\ln(x)}{a^4}+\left(\sum_{\_R=\text{RootOf}((64a^7c^3-48a^6b^2c^2+12a^5b^4c-a^4b \cdots}\right.$

input
```
int(x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2/a^3*b/x+1/a^4*((a*c*b*(3*a*c-b^2)/
(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*
a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*ln(c*x^2+b*x+a)+2*(19*a^2
*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(194) = 388$.

Time = 0.45 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.07

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2
+ 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c
^3)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4
*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)
*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7
 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*
c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x
^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c
 - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*
b^2*c^2 - 32*a^4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 3
0*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c...
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

3.26.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = -\frac{\left(3\,b^5-20\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}} - \frac{\left(3\,b^2-2\,ac\right)\log\left(cx^2+bx+a\right)}{2\,a^4} + \frac{\left(3\,b^2-2\,ac\right)\log\left(|x|\right)}{a^4} - \frac{a^3b^2-4\,a^4c-2\,\left(3\,ab^3c-11\,a^2bc^2\right)x^3-\left(6\,ab^4-25\,a^2b^2c+8\,a^3c^2\right)x^2-3\,\left(a^2b^3-4\,a^3bc\right)x}{2\,\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)a^4x^2}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c
 - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)
*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2
)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.52

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^2} \, dx$$
$$= \frac{\ln\left(6\,a\,b^8 + 6\,b^9\,x + 192\,a^5\,c^4 - 6\,a\,b^5\,\sqrt{-(4\,a\,c - b^2)^3} - 73\,a^2\,b^6\,c - 6\,b^6\,x\,\sqrt{-(4\,a\,c - b^2)^3} + 307\,a^3\,b^4\,c^2 \right.}{}$$
$$- \frac{\ln(x)\,(2\,a\,c - 3\,b^2)}{a^4} - \frac{\frac{1}{2\,a} - \frac{3\,b\,x}{2\,a^2} + \frac{x^2\,(8\,a^2\,c^2 - 25\,a\,b^2\,c + 6\,b^4)}{2\,a^3\,(4\,a\,c - b^2)} - \frac{b\,c\,x^3\,(11\,a\,c - 3\,b^2)}{a^3\,(4\,a\,c - b^2)}}{c\,x^4 + b\,x^3 + a\,x^2}$$
$$+ \frac{\ln\left(6\,a\,b^8 + 6\,b^9\,x + 192\,a^5\,c^4 + 6\,a\,b^5\,\sqrt{-(4\,a\,c - b^2)^3} - 73\,a^2\,b^6\,c + 6\,b^6\,x\,\sqrt{-(4\,a\,c - b^2)^3} + 307\,a^3\,b^4\,c \right.}{}$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
(log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) -
73*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^
4*b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(
4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4
*a*c - b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 +
 128*a^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^
2))/a^4 - (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*
c))/(2*a^3*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2))
)/(a*x^2 + b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5
*(-(4*a*c - b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2
) + 307*a^3*b^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1
/2) + 27*a^3*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*
b^3*c^3*x - 24*a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4
*b*c^4*x - 40*a*b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a
*c - b^2)^3)^(1/2))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - ...
```

3.27 $\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx$

3.27.1 Optimal result

Integrand size = 18, antiderivative size = 252

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{2(2b^2-5ac)}{3a^2(b^2-4ac)x^3}+\frac{b(2b^2-7ac)}{a^3(b^2-4ac)x^2} - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4(b^2-4ac)x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)x^3(a+bx+cx^2)} - \frac{2(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}} - \frac{2b(2b^2-3ac)\log(x)}{a^5}+\frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5}$$

output
```
-2/3*(-5*a*c+2*b^2)/a^2/(-4*a*c+b^2)/x^3+b*(-7*a*c+2*b^2)/a^3/(-4*a*c+b^2)
/x^2-2*(5*a^2*c^2-9*a*b^2*c+2*b^4)/a^4/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/
(-4*a*c+b^2)/x^3/(c*x^2+b*x+a)-2*(-10*a^3*c^3+30*a^2*b^2*c^2-15*a*b^4*c+2*
b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^5/(-4*a*c+b^2)^(3/2)-2*b*(-3*
a*c+2*b^2)*ln(x)/a^5+b*(-3*a*c+2*b^2)*ln(c*x^2+b*x+a)/a^5
```

3.27.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \frac{-\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3a(-3b^2+2ac)}{x}-\frac{3a(b^5-5ab^3c+5a^2bc^2+b^4cx-4ab^2c^2x+2a^2c^3x)}{(b^2-4ac)(a+x(b+cx))}-\frac{6(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4a}}\right.}{(-b^2+4ac)^{3/2}}}{3a^5}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

output
```
(-(a^3/x^3) + (3*a^2*b)/x^2 + (3*a*(-3*b^2 + 2*a*c))/x - (3*a*(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 + b^4*c*x - 4*a*b^2*c^2*x + 2*a^2*c^3*x))/((b^2 - 4*a*c
)*(a + x*(b + c*x))) - (6*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^
3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 6*(-2*b^
3 + 3*a*b*c)*Log[x] + 3*(2*b^3 - 3*a*b*c)*Log[a + x*(b + c*x)])/(3*a^5)
```

3.27.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1949, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$\Big\downarrow \text{1949}$$
$$\int \frac{1}{x^4(a+bx+cx^2)^2}dx$$
$$\Big\downarrow \text{1165}$$
$$\frac{-2ac+b^2+bcx}{ax^3(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{2(2b^2+2cxb-5ac)}{x^4(cx^2+bx+a)}dx}{a(b^2-4ac)}$$
$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{2b^2+2cxb-5ac}{x^4(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^3(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 1200$$

$$\frac{2\int \left(\frac{2b^2-5ac}{ax^4}+\frac{b(b^2-4ac)(3ac-2b^2)}{a^4x}+\frac{2b^6-13acb^4+21a^2c^2b^2+c(b^2-4ac)(2b^2-3ac)xb-5a^3c^3}{a^4(cx^2+bx+a)}+\frac{2b^4-9acb^2+5a^2c^2}{a^3x^2}+\frac{b(7ac-2b^2)}{a^2x^3}\right)dx}{a(b^2-4ac)}$$

$$\frac{-2ac+b^2+bcx}{ax^3(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{b(b^2-4ac)(2b^2-3ac)\log(a+bx+cx^2)}{2a^4}-\frac{b\log(x)(b^2-4ac)(2b^2-3ac)}{a^4}+\frac{b(2b^2-7ac)}{2a^2x^2}-\frac{5a^2c^2-9ab^2c+2b^4}{a^3x}-\frac{(-10a^3c^3+30a^2b^2c^2-15ab^4}{a^4\sqrt{b}}\right.}{a(b^2-4ac)}$$

$$\frac{-2ac+b^2+bcx}{ax^3(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) + (2*(-1/3*(
2*b^2 - 5*a*c)/(a*x^3) + (b*(2*b^2 - 7*a*c))/(2*a^2*x^2) - (2*b^4 - 9*a*b^
2*c + 5*a^2*c^2)/(a^3*x) - ((2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*
c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - (b*
(b^2 - 4*a*c)*(2*b^2 - 3*a*c)*Log[x])/a^4 + (b*(b^2 - 4*a*c)*(2*b^2 - 3*a*
c)*Log[a + b*x + c*x^2])/(2*a^4)))/(a*(b^2 - 4*a*c))
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.17

method	result
default	$-\frac{1}{3a^2x^3}-\frac{-2ac+3b^2}{x\,a^4}+\frac{b}{a^3x^2}+\frac{2b(3ac-2b^2)\ln(x)}{a^5}+\frac{\frac{ac\left(2a^2c^2-4a\,b^2c+b^4\right)x}{4ac-b^2}+\frac{ab\left(5a^2c^2-5a\,b^2c+b^4\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\left(-12a^2b\,c^3+11a\,b^3c^2-2b\right.}{c}$
risch	$\frac{\frac{2c\left(5a^2c^2-9a\,b^2c+2b^4\right)x^4}{\left(4ac-b^2\right)a^4}+\frac{b\left(17a^2c^2-20a\,b^2c+4b^4\right)x^3}{a^4\left(4ac-b^2\right)}+\frac{\left(5ac-6b^2\right)x^2}{3a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(c\,x^2+bx+a)}+\frac{6b\ln(x)c}{a^4}-\frac{4b^3\ln(x)}{a^5}+2\left(\sum_{\_R=\mathrm{RootOf}\left((64c^3\right.}\right.$

input
```
int(1/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3-(-2*a*c+3*b^2)/x/a^4+1/a^3*b/x^2+2*b*(3*a*c-2*b^2)/a^5*ln(x)+
1/a^5*((a*c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x+a*b*(5*a^2*c^2-5*a*b^2
*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-12*a^2*b*c^3+11*a*
b^3*c^2-2*b^5*c)/c*ln(c*x^2+b*x+a)+2*(5*c^3*a^3-21*a^2*b^2*c^2+13*a*b^4*c-
2*b^6-1/2*(-12*a^2*b*c^3+11*a*b^3*c^2-2*b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(246) = 492$.

Time = 0.56 (sec) , antiderivative size = 1407, normalized size of antiderivative = 5.58

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^2}\, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2
+ 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^
3*c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 -
80*a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4
)*x^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^
6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log
((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c -
 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c +
 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b
^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*
c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^
4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 4
8*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 +
(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a
^8*c^2)*x^3), -1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17
*a^2*b^4*c^2 + 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*
c + 97*a^3*b^3*c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a
^4*b^2*c^2 - 80*a^5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3
 - 10*a^3*c^4)*x^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^...
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

3.27.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,(2\,b^6-15\,ab^4c+30\,a^2b^2c^2-10\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^5b^2-4\,a^6c)\sqrt{-b^2+4\,ac}} + \frac{(2\,b^3-3\,abc)\log\left(cx^2+bx+a\right)}{a^5} - \frac{2\,(2\,b^3-3\,abc)\log\left(|x|\right)}{a^5} - \frac{a^4b^2-4\,a^5c+6\,(2\,ab^4c-9\,a^2b^2c^2+5\,a^3c^3)x^4+3\,(4\,ab^5-20\,a^2b^3c+17\,a^3bc^2)x^3+(6\,a^2b^4-29\,a^3b^2c+}{3\,(cx^2+bx+a)(b^2-4\,ac)a^5x^3}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c)) + (2*b^3 - 3*a*
b*c)*log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*log(abs(x))/a^5 - 1/3*
(a^4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*
a*b^5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*
a^4*c^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)
*a^5*x^3)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 1120, normalized size of antiderivative = 4.44

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{x^2\,\left(5\,a\,c-6\,b^2\right)}{3\,a^3}-\frac{1}{3\,a}+\frac{2\,b\,x}{3\,a^2}+\frac{x^3\,\left(17\,a^2\,b\,c^2-20\,a\,b^3\,c+4\,b^5\right)}{a^4\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,x^4\,\left(5\,a^2\,c^2-9\,a\,b^2\,c+2\,b^4\right)}{a^4\,\left(4\,a\,c-b^2\right)}}{c\,x^5+b\,x^4+a\,x^3}$$
$$+\frac{\ln\left(4\,a\,b^9+4\,b^{10}\,x-4\,a\,b^6\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-52\,a^2\,b^7\,c+308\,a^5\,b\,c^4-40\,a^5\,c^5\,x-4\,b^7\,x\,\sqrt{-{\left(4\,a\,c-b^2\right.}}\right.}{}$$
$$+\frac{\ln\left(4\,a\,b^9+4\,b^{10}\,x+4\,a\,b^6\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-52\,a^2\,b^7\,c+308\,a^5\,b\,c^4-40\,a^5\,c^5\,x+4\,b^7\,x\,\sqrt{-{\left(4\,a\,c-b^2\right.}}\right.}{}$$
$$+\frac{2\,b\,\ln\left(x\right)\,\left(3\,a\,c-2\,b^2\right)}{a^5}$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
((x^2*(5*a*c - 6*b^2))/(3*a^3) - 1/(3*a) + (2*b*x)/(3*a^2) + (x^3*(4*b^5 +
 17*a^2*b*c^2 - 20*a*b^3*c))/(a^4*(4*a*c - b^2)) + (2*c*x^4*(2*b^4 + 5*a^2
*c^2 - 9*a*b^2*c))/(a^4*(4*a*c - b^2)))/(a*x^3 + b*x^4 + c*x^5) + (log(4*a
*b^9 + 4*b^10*x - 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) - 52*a^2*b^7*c + 308*a^
5*b*c^4 - 40*a^5*c^5*x - 4*b^7*x*(-(4*a*c - b^2)^3)^(1/2) + 243*a^3*b^5*c^
2 - 473*a^4*b^3*c^3 + 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^4*c*(-
(4*a*c - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4*c^3*x + 438*a^4*b
^2*c^4*x - 54*a*b^8*c*x - 33*a^3*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a*b
^5*c*x*(-(4*a*c - b^2)^3)^(1/2) + 41*a^3*b*c^3*x*(-(4*a*c - b^2)^3)^(1/2)
- 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*c^2 - 30*b^2*c^
2*(-(4*a*c - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 - 10*c^3*(-(4*a*c - b^2)^3)
^(1/2)) + 2*b^9 - 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*c - 15*b^4*c*
(-(4*a*c - b^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*c^3 - 12*a^6
*b^4*c + 48*a^7*b^2*c^2) + (log(4*a*b^9 + 4*b^10*x + 4*a*b^6*(-(4*a*c - b^
2)^3)^(1/2) - 52*a^2*b^7*c + 308*a^5*b*c^4 - 40*a^5*c^5*x + 4*b^7*x*(-(4*a
*c - b^2)^3)^(1/2) + 243*a^3*b^5*c^2 - 473*a^4*b^3*c^3 - 5*a^4*c^3*(-(4*a*
c - b^2)^3)^(1/2) - 24*a^2*b^4*c*(-(4*a*c - b^2)^3)^(1/2) + 266*a^2*b^6*c^
2*x - 563*a^3*b^4*c^3*x + 438*a^4*b^2*c^4*x - 54*a*b^8*c*x + 33*a^3*b^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 30*a*b^5*c*x*(-(4*a*c - b^2)^3)^(1/2) - 41*a^
3*b*c^3*x*(-(4*a*c - b^2)^3)^(1/2) + 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3...
```

3.27. $\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx$

3.28 $\int \frac{1}{x(ax^2+bx^3+cx^4)^2}\,dx$

3.28.1 Optimal result

Integrand size = 22, antiderivative size = 318

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3}-\frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2}+\frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)}+\frac{b(5b^6-42ab^4c+105a^2b^2c^2-70a^3c^3)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}}+\frac{(5b^4-12ab^2c+3a^2c^2)\log(x)}{a^6}-\frac{(5b^4-12ab^2c+3a^2c^2)\log\left(a+bx+cx^2\right)}{2a^6}$$

output
```
1/4*(12*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^4+1/3*b*(-17*a*c+5*b^2)/a^3/(-4*a*c+
b^2)/x^3+1/2*(-12*a^2*c^2+22*a*b^2*c-5*b^4)/a^4/(-4*a*c+b^2)/x^2+b*(29*a^2
*c^2-27*a*b^2*c+5*b^4)/a^5/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)
/x^4/(c*x^2+b*x+a)+b*(-70*a^3*c^3+105*a^2*b^2*c^2-42*a*b^4*c+5*b^6)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^6/(-4*a*c+b^2)^(3/2)+(3*a^2*c^2-12*a*b^2
*c+5*b^4)*ln(x)/a^6-1/2*(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(c*x^2+b*x+a)/a^6
```

3.28.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(ax^2+bx^3+cx^4)^2}\,dx$$

$$= \frac{-\frac{3a^4}{x^4}+\frac{8a^3b}{x^3}+\frac{6a^2(-3b^2+2ac)}{x^2}-\frac{24ab(-2b^2+3ac)}{x}-\frac{12a(-b^6+6ab^4c-9a^2b^2c^2+2a^3c^3-b^5cx+5ab^3c^2x-5a^2bc^3x)}{(b^2-4ac)(a+x(b+cx))}+\frac{12b(5b^6-42ab^4c}{}}{}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

output
```
((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (24*a*b*(-2
*b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c^2 + 2*a^3*c^3 - b
^5*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (12*b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2
*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c +
 3*a^2*c^2)*Log[x] - 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x
)])/(12*a^6)
```

3.28.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(ax^2+bx^3+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^5(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1165$$

$$\frac{-2ac+b^2+bcx}{ax^4(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{5b^2+5cxb-12ac}{x^5(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5b^2+5cxb-12ac}{x^5(cx^2+bx+a)}dx}{a(b^2-4ac)} + \frac{-2ac+b^2+bcx}{ax^4(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \ 1200$$

$$\frac{\int \left(\frac{5b^2-12ac}{ax^5} + \frac{(b^2-4ac)(5b^4-12acb^2+3a^2c^2)}{a^5x} + \frac{-b(5b^6-37acb^4+78a^2c^2b^2-41a^3c^3)-c(b^2-4ac)(5b^4-12acb^2+3a^2c^2)x}{a^5(cx^2+bx+a)} + \frac{-5b^5+27acb^3-}{a^4x^2}\right.}{a(b^2-4ac)}$$

$$\frac{-2ac+b^2+bcx}{ax^4(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{b(5b^2-17ac)}{3a^2x^3} - \frac{(b^2-4ac)(3a^2c^2-12ab^2c+5b^4)\log(a+bx+cx^2)}{2a^5} + \frac{\log(x)(b^2-4ac)(3a^2c^2-12ab^2c+5b^4)}{a^5} + \frac{b(29a^2c^2-27ab^2c+5b^4)}{a^4x} - \frac{12a^2c^2}{}}{a(b^2-4ac)}$$

$$\frac{-2ac+b^2+bcx}{ax^4(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (-1/4*(5*b
^2 - 12*a*c)/(a*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^2*x^3) - (5*b^4 - 22*a*b^
2*c + 12*a^2*c^2)/(2*a^3*x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^4
*x) + (b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(5*b^4
 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/a^5 - ((b^2 - 4*a*c)*(5*b^4 - 12*a*b^2*
c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^5))/(a*(b^2 - 4*a*c))
```

3.28.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.28.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.13

method	result
default	$-\frac{1}{4a^2x^4}-\frac{-2ac+3b^2}{2x^2a^4}+\frac{(3a^2c^2-12a\,b^2c+5b^4)\ln(x)}{a^6}+\frac{2b}{3a^3x^3}-\frac{2b(3ac-2b^2)}{a^5x}-\frac{\frac{acb\left(5a^2c^2-5a\,b^2c+b^4\right)x}{4ac-b^2}-\frac{a\left(2c^3a^3-9a^2b^2c^2\cdots\right.}{4ac-b^2}}{c\,x^2+bx+a}$
risch	Expression too large to display

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/x^4-1/2*(-2*a*c+3*b^2)/x^2/a^4+(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(x)
/a^6+2/3/a^3*b/x^3-2*b*(3*a*c-2*b^2)/a^5/x-1/a^6*((a*c*b*(5*a^2*c^2-5*a*b^
2*c+b^4)/(4*a*c-b^2)*x-a*(2*a^3*c^3-9*a^2*b^2*c^2+6*a*b^4*c-b^6)/(4*a*c-b^
2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(12*a^3*c^4-51*a^2*b^2*c^3+32*a*b^4*c
^2-5*b^6*c)/c*ln(c*x^2+b*x+a)+2*(41*b*c^3*a^3-78*b^3*c^2*a^2+37*b^5*c*a-5*
b^7-1/2*(12*a^3*c^4-51*a^2*b^2*c^3+32*a*b^4*c^2-5*b^6*c)*b/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(306) = 612$.

Time = 0.73 (sec) , antiderivative size = 1640, normalized size of antiderivative = 5.16

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5
*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c +
 316*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146
*a^3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b
^4*c + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 1
05*a^2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2
 - 70*a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^
4*b*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 5*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a
^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216
*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c
^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) - 12*((5*b^8*
c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (
5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5
 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^
4)*x^4)*log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 -
 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^
4), -1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*
b^5*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b...
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

3.28.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = -\frac{\left(5\,b^7-42\,ab^5c+105\,a^2b^3c^2-70\,a^3bc^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^6b^2-4\,a^7c\right)\sqrt{-b^2+4\,ac}} - \frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\log\left(cx^2+bx+a\right)}{2\,a^6} + \frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\log\left(|x|\right)}{a^6} - \frac{3\,a^5b^2-12\,a^6c-12\left(5\,ab^5c-27\,a^2b^3c^2+29\,a^3bc^3\right)x^5-6\left(10\,ab^6-59\,a^2b^4c+80\,a^3b^2c^2-12\,a^4c^3\right)x^4-\cdots}{12\left(cx^2+bx+a\right)\left(b^2-\cdots\right.}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/((a^6*b^2 - 4*a^7*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(5*b^4
- 12*a*b^2*c + 3*a^2*c^2)*log(c*x^2 + b*x + a)/a^6 + (5*b^4 - 12*a*b^2*c +
 3*a^2*c^2)*log(abs(x))/a^6 - 1/12*(3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c -
 27*a^2*b^3*c^2 + 29*a^3*b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*
b^2*c^2 - 12*a^4*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*
x^3 + (10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*
c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 1260, normalized size of antiderivative = 3.96

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \frac{\ln(x)\,\left(3\,a^2\,c^2-12\,a\,b^2\,c+5\,b^4\right)}{a^6} - \frac{\frac{1}{4\,a}-\frac{x^2\,\left(9\,a\,c-10\,b^2\right)}{12\,a^3}-\frac{5\,b\,x}{12\,a^2}+\frac{x^4\,\left(-12\,a^3\,c^3+80\,a^2\,b^2\,c^2-59\,a\,b^4\,c+10\,b^6\right)}{2\,a^5\,\left(4\,a\,c-b^2\right)}+\frac{b\,x^3\,\left(26\,a\,c-15\,b^2\right)}{6\,a^4}+\frac{b\,c\,x^5\,\left(29\,a^2\,c^2-27\,a\,b^2\,c+5\,b^4\right)}{a^5\,\left(4\,a\,c-b^2\right)}}{c\,x^6+b\,x^5+a\,x^4} + \frac{\ln\left(288\,a^6\,c^5-10\,b^{11}\,x-10\,a\,b^{10}+10\,a\,b^7\,\sqrt{-\left(4\,a\,c-b^2\right)^3}+139\,a^2\,b^8\,c+10\,b^8\,x\,\sqrt{-\left(4\,a\,c-b^2\right)^3}-71}{} - \frac{\ln\left(10\,a\,b^{10}+10\,b^{11}\,x-288\,a^6\,c^5+10\,a\,b^7\,\sqrt{-\left(4\,a\,c-b^2\right)^3}-139\,a^2\,b^8\,c+10\,b^8\,x\,\sqrt{-\left(4\,a\,c-b^2\right)^3}+71}{}$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x)
```

output
```
(log(x)*(5*b^4 + 3*a^2*c^2 - 12*a*b^2*c))/a^6 - (1/(4*a) - (x^2*(9*a*c - 1
0*b^2))/(12*a^3) - (5*b*x)/(12*a^2) + (x^4*(10*b^6 - 12*a^3*c^3 + 80*a^2*b
^2*c^2 - 59*a*b^4*c))/(2*a^5*(4*a*c - b^2)) + (b*x^3*(26*a*c - 15*b^2))/(6
*a^4) + (b*c*x^5*(5*b^4 + 29*a^2*c^2 - 27*a*b^2*c))/(a^5*(4*a*c - b^2)))/(
a*x^4 + b*x^5 + c*x^6) + (log(288*a^6*c^5 - 10*b^11*x - 10*a*b^10 + 10*a*b
^7*(-(4*a*c - b^2)^3)^(1/2) + 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^
(1/2) - 717*a^3*b^6*c^2 + 1643*a^4*b^4*c^3 - 1508*a^5*b^2*c^4 - 69*a^2*b^5
*c*(-(4*a*c - b^2)^3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) - 779*
a^2*b^7*c^2*x + 1916*a^3*b^5*c^3*x - 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(
4*a*c - b^2)^3)^(1/2) + 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 568*a^5*b*c^5*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2
*b^4*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)
^(1/2))*(a^3*(466*b^4*c^3 - 35*b*c^3*(-(4*a*c - b^2)^3)^(1/2)) - a^2*((387
*b^6*c^2)/2 - (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - (5*b^10)/2 + 96*
a^5*c^5 + (5*b^7*(-(4*a*c - b^2)^3)^(1/2))/2 + a*(36*b^8*c - 21*b^5*c*(-(4
*a*c - b^2)^3)^(1/2)) - 456*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2) - (log(10*a*b^10 + 10*b^11*x - 288*a^6*c^5 + 10*a*b
^7*(-(4*a*c - b^2)^3)^(1/2) - 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^
(1/2) + 717*a^3*b^6*c^2 - 1643*a^4*b^4*c^3 + 1508*a^5*b^2*c^4 - 69*a^2*b^5
*c*(-(4*a*c - b^2)^3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) + 7...
```

3.29 $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

3.29.1 Optimal result

Integrand size = 24, antiderivative size = 257

$$\begin{aligned}\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx &= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}\\&\quad-\frac{(7b^2-16ac)\,x\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}\\&\quad+\frac{b(7b^2-12ac)\,(b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

output
```
1/256*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^
2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/96
0*b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/1920*(256*a^2*c^2-46
0*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/240*(-16*a*c+7*b^2)*x
*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/40*x^2*(8*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2
)/c
```

3.29.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.72

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{2\sqrt{c}x(a+x(b+cx))\left(-105b^4+70b^3cx+4b^2c(115a-14cx^2)+8bc^2x(-29a+6cx^2)+128c^2(-2a^2+acx\right.}{3840c^{9/2}\sqrt{x^2(a}}$$

input
```
Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 1
4*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2
*x^4)) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Lo
g[c^4*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(3840*c^(9/2)*Sqrt[x
^2*(a + x*(b + c*x))])
```

3.29.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1966, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$\Big\downarrow \text{1966}$$
$$\frac{\int -\frac{x^3(6ab+(7b^2-16ac)x)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{40c}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}$$
$$\Big\downarrow \text{27}$$
$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\int \frac{x^3(6ab+(7b^2-16ac)x)}{\sqrt{cx^4+bx^3+ax^2}}dx}{80c}$$
$$\Big\downarrow \text{1996}$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int \frac{x^2\left(4a\left(7b^2-16ac\right)+b\left(35b^2-116ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{3c}}{80c}$$

$$\downarrow \; 27$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int \frac{x^2\left(4a\left(7b^2-16ac\right)+b\left(35b^2-116ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{6c}}{80c}$$

$$\downarrow \; 1996$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab\left(35b^2-116ac\right)+\left(105b^4-460acb^2+256a^2c^2\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{6c}}{80c}$$

$$\downarrow \; 27$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab\left(35b^2-116ac\right)+\left(105b^4-460acb^2+256a^2c^2\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}}{6c}}{80c}$$

$$\downarrow \; 1996$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}}{6c}}{80c}$$

$$\downarrow \; 27$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3+a\cdots}}\cdots}{2c}}{4c}}{6c}}{80c}$$

$$\downarrow \; 1961$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15bx\left(7b^2-12ac\right)\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int\cdots}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}}{80c}$$

3.29. $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

$$\Big\downarrow \; 1092$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\frac{15bx(7b^2-12ac)(b^2-4ac)\sqrt{a+bx+cx^2}\int}{c\sqrt{ax^2+bx^3+}}}{4c}}{6c}}{80c}$$

$$\Big\downarrow \; 219$$

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\frac{15bx(7b^2-12ac)(b^2-4ac)\sqrt{a+bx+cx^2}\text{a}}{2c^{3/2}\sqrt{ax^2+bx^3}}}{4c}}{6c}}{80c}$$

input
```
Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x^2*(b + 8*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*c) - ((((7*b^2 - 16*a*c)*
x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c) - ((b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(2*c) - (((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(c*x) - (15*b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sqrt[a
+ b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c))/(6*c))/(80*c)
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.29. $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

3.29.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{45}{32}a^2 b\,c^2+\frac{75}{64}a\,b^3 c-\frac{105}{512}b^5\right)\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)+\sqrt{c\,x^2+bx+a}\left(\left(\frac{7}{32}b^2x^2+\frac{29}{32}abx+a^2\right)c^{\frac{5}{2}}-\frac{115\left(\frac{7bx}{46}+a\right)b}{64}\right.\right.}{15c^{\frac{9}{2}}}$
risch	$-\frac{\left(-384c^4x^4-48b\,c^3x^3-128a\,c^3x^2+56b^2c^2x^2+232ab\,c^2x-70b^3cx+256a^2c^2-460a\,b^2c+105b^4\right)\sqrt{x^2(c\,x^2+bx+a)}}{1920c^4x}+\frac{b(48c}{}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(768x^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{9}{2}}-672c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}bx-512c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}a+720c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,abx+56\right.}{}$

input
```
int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/c^(9/2)*((-45/32*a^2*b*c^2+75/64*a*b^3*c-105/512*b^5)*ln(2*(c*x^2+b*
x+a)^(1/2)*c^(1/2)+2*c*x+b)+(c*x^2+b*x+a)^(1/2)*((7/32*b^2*x^2+29/32*a*b*x
+a^2)*c^(5/2)-115/64*(7/46*b*x+a)*b^2*c^(3/2)-1/2*(3/8*b*x+a)*x^2*c^(7/2)-
3/2*c^(9/2)*x^4+105/256*c^(1/2)*b^4))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.52

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(384\,c^5x^4+48\right.}{768}\right.$$
$$-\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\left(384\,c^5x^4+48\,bc^4x^3-105\,b^4c\right.}{3840\,c^5x}$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3
+ 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4
*a*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 -
256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*
x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c +
 48*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 -
 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 +
2*(35*b^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

3.29.6 Sympy [F]

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int x^2\sqrt{x^2\left(a+bx+cx^2\right)}\,dx$$

input
```
integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.29.7 Maxima [F]

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int\sqrt{cx^4+bx^3+ax^2}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.07

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{7\,b^2c^2\mathrm{sgn}(x)-16\,ac^3\mathrm{sgn}(x)}{c^4}\right)x+\frac{35\,b^3c\mathrm{sgn}(x)}{}\right.\right.$$
$$-\frac{(7\,b^5\mathrm{sgn}(x)-40\,ab^3c\mathrm{sgn}(x)+48\,a^2bc^2\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{9}{2}}}$$
$$+\frac{\left(105\,b^5\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-600\,ab^3c\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+720\,a^2bc^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+210\sqrt{a}b^4\sqrt{}\right.}{3840\,c^{\frac{9}{2}}}$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2
*c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn
(x))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c
^4) - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(a
bs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2) + 1/3840*(1
05*b^5*log(abs(b - 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(b - 2*sqrt(a)
*sqrt(c))) + 720*a^2*b*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b
^4*sqrt(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))*sgn(x)/c^(9/2)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

input
```
int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.30 $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

3.30.1 Optimal result

Integrand size = 22, antiderivative size = 205

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{(b^2-4ac)(5b^2-4ac)x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/96*(
-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/192*b*(-52*a*c+15*b^2)*(c*x
^4+b*x^3+a*x^2)^(1/2)/c^3/x+1/24*x*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
```

3.30.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.73

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{2\sqrt{c}x(a+x(b+cx))\left(15b^3-10b^2cx+24c^2x(a+2cx^2)+b(-52ac+8c^2x^2)\right)+3(5b^4-24ab^2c+16a^2c^2)}{384c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^
2) + b*(-52*a*c + 8*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt
[a + x*(b + c*x)]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c
^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.30.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1966, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$\Big\downarrow \text{1966}$$
$$\frac{\int -\frac{x^2(4ab+(5b^2-12ac)x)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{24c}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}$$
$$\Big\downarrow \text{27}$$
$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\int \frac{x^2(4ab+(5b^2-12ac)x)}{\sqrt{cx^4+bx^3+ax^2}}dx}{48c}$$
$$\Big\downarrow \text{1996}$$
$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{48c}$$

$$\Big\downarrow \; 27$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}}{48c}$$

$$\Big\downarrow \; 1996$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}}{48c}$$

$$\Big\downarrow \; 27$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}}{48c}$$

$$\Big\downarrow \; 1961$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{48c}$$

$$\Big\downarrow \; 1092$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{48c}$$

$$\Big\downarrow \; 219$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}}{48c}$$

input
```
Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

3.30. $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

output
```
(x*(b + 6*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c) - (((5*b^2 - 12*a*c)*Sq
rt[a*x^2 + b*x^3 + c*x^4])/(2*c) - ((b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^
3 + c*x^4])/(c*x) - (3*(b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^
2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt
[a*x^2 + b*x^3 + c*x^4]))/(4*c))/(48*c)
```

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$-\frac{\left(a^{2} c^{2}-\frac{3}{2} a\,b^{2} c+\frac{5}{16} b^{4}\right) \ln \left(2 \sqrt{c\,x^{2}+b x+a}\,\sqrt{c}+2 c x+b \right)+\frac{13 \sqrt{c\,x^{2}+b x+a}\,\left(b \left(\frac{5 b x}{26}+a \right) c^{\frac{3}{2}}-\frac{6 \left(\frac{b x}{3}+a \right) x\,c^{\frac{5}{2}}}{13}-\frac{15 \sqrt{c}\,b^{3}}{52}-\frac{12 c^{\frac{7}{2}} x^{3}}{13}\right)}{6}}{8 c^{\frac{7}{2}}}$
risch	$-\frac{\left(-48 c^{3} x^{3}-8 b\,c^{2} x^{2}-24 a\,c^{2} x+10 b^{2} c x+52 a b c-15 b^{3}\right) \sqrt{x^{2} \left(c\,x^{2}+b x+a \right)}}{192 c^{3} x}-\frac{\left(16 a^{2} c^{2}-24 a\,b^{2} c+5 b^{4}\right) \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+\right.}$ … $\frac{}{128 c^{\frac{7}{2}} x \sqrt{c\,x^{2}+b x+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(96 x \left(c\,x^{2}+b x+a \right)^{\frac{3}{2}} c^{\frac{7}{2}}-48 c^{\frac{7}{2}} \sqrt{c\,x^{2}+b x+a}\,a x-80 c^{\frac{5}{2}} \left(c\,x^{2}+b x+a \right)^{\frac{3}{2}} b+60 c^{\frac{5}{2}} \sqrt{c\,x^{2}+b x+a}\,b^{2} x-24 c^{\frac{5}{2}} \sqrt{c\,x\right.}{}$

input
```
int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c^(7/2)*((a^2*c^2-3/2*a*b^2*c+5/16*b^4)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1
/2)+2*c*x+b)+13/6*(c*x^2+b*x+a)^(1/2)*(b*(5/26*b*x+a)*c^(3/2)-6/13*(1/3*b*
x+a)*x*c^(5/2)-15/52*c^(1/2)*b^3-12/13*c^(7/2)*x^3))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.59

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$= \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,(48\,c^4x^3+8\,bc^3x}{768\,c^4x}\right.$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*
b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c
)*x)/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c
^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 -
24*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)
*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(48*c^4*x^3 + 8*b*c
^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 +
b*x^3 + a*x^2))/(c^4*x)]
```

3.30.6 Sympy [F]

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx = \int x\sqrt{x^2\,(a+bx+cx^2)}\,dx$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.30.7 Maxima [F]

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int \sqrt{cx^4+bx^3+ax^2}x\,dx$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,x\text{sgn}(x)+\frac{b\text{sgn}(x)}{c}\right)x-\frac{5\,b^2c\text{sgn}(x)-12\,ac^2\text{sgn}(x)}{c^3}\right)x+\frac{15\,b^3\text{sgn}(x)-52\,a}{c^3}\right.$$
$$+\frac{(5\,b^4\text{sgn}(x)-24\,ab^2c\text{sgn}(x)+16\,a^2c^2\text{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^3\sqrt{c}-10\right.}{384\,c^{\frac{7}{2}}}$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*sgn(x) + b*sgn(x)/c)*x - (5*b^2*c*s
gn(x) - 12*a*c^2*sgn(x))/c^3)*x + (15*b^3*sgn(x) - 52*a*b*c*sgn(x))/c^3) +
 1/128*(5*b^4*sgn(x) - 24*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2) - 1/384*(15*b^4*lo
g(abs(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c)))
 + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 1
04*a^(3/2)*b*c^(3/2))*sgn(x)/c^(7/2)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.31 $\int \sqrt{ax^2+bx^3+cx^4}\,dx$

3.31.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx=-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx}+\frac{b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}$$

output
```
-1/8*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x+1/3*(c*x^2+b*x+a)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c/x+1/16*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))*(c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2)/x/(c*x^2+b*x+a)^(1/2
)
```

3.31.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx =\frac{2\sqrt{c}x(a+x(b+cx))\left(-3b^2+2bcx+8c(a+cx^2)\right)-3(b^3-4abc)\,x\sqrt{a+x(b+cx)}\log\left(c^2\left(b+2cx-2\sqrt{\right.\right.}}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2)) - 3*(b
^3 - 4*a*b*c)*x*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])])/(48*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.31.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1950, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx$$

$$\Big\downarrow \text{ 1950}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\int x\sqrt{cx^2+bx+a}dx}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\int \sqrt{cx^2+bx+a}dx}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(Sqrt[a*x^2 + b*x^3 + c*x^4]*((a + b*x + c*x^2)^(3/2)/(3*c) - (b*(((b + 2*
c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(2*c)))/(x*Sqrt[a + b*x + c
*x^2])
```

3.31.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1950
```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Simp[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))])   Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))],
x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

3.31.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(8c^2x^2+2bcx+8ac-3b^2)\sqrt{x^2(cx^2+bx+a)}}{24c^2x}-\frac{b(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\sqrt{x^2(cx^2+bx+a)}}{16c^{\frac{5}{2}}x\sqrt{cx^2+bx+a}}$
pseudoelliptic	$\frac{16x^2\sqrt{cx^2+bx+a}\,c^{\frac{5}{2}}+4c^{\frac{3}{2}}\sqrt{cx^2+bx+a}\,bx+16a\,c^{\frac{3}{2}}\sqrt{cx^2+bx+a}-6\sqrt{c}\sqrt{cx^2+bx+a}\,b^2-12\ln\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b\right)a}{48c^{\frac{5}{2}}}$
default	$\frac{\sqrt{cx^4+bx^3+ax^2}\left(16(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{cx^2+bx+a}\,bx-6c^{\frac{3}{2}}\sqrt{cx^2+bx+a}\,b^2-12\ln\left(\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b}{2\sqrt{c}}\right)ab\,c^2-\right.}{48x\sqrt{cx^2+bx+a}\,c^{\frac{7}{2}}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*x^2+2*b*c*x+8*a*c-3*b^2)/c^2*(x^2*(c*x^2+b*x+a))^(1/2)/x-1/16*
b*(4*a*c-b^2)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*(x^2*(c*
x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.60

$$\int\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\left[-\frac{3\,(b^3-4\,abc)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,(8\,c^3x^2+2\,bc^2x-3\,b^2c+8\,c}{96\,c^3x}\right.$$
$$-\frac{3\,(b^3-4\,abc)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(8\,c^3x^2+2\,bc^2x-3\,b^2c+8\,ac^2)\sqrt{cx^4+bx^3+}}{48\,c^3x}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(8*c^3
*x^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)
, -1/48*(3*(b^3 - 4*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(8*c^3*x^2 + 2*b*
c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]
```

3.31.6 Sympy [F]

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx=\int \sqrt{ax^2+bx^3+cx^4}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

3.31.7 Maxima [F]

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx=\int \sqrt{cx^4+bx^3+ax^2}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \sqrt{ax^2+bx^3+cx^4}\,dx &= \frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(4\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{3\,b^2\mathrm{sgn}(x)-8\,ac\mathrm{sgn}(x)}{c^2}\right)\\ &\quad-\frac{\left(b^3\mathrm{sgn}(x)-4\,abc\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}\\ &\quad+\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{48\,c^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sgn(x) + b*sgn(x)/c)*x - (3*b^2*sgn(x)
- 8*a*c*sgn(x))/c^2) - 1/16*(b^3*sgn(x) - 4*a*b*c*sgn(x))*log(abs(2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) + 1/48*(3*b^3*log(abs(
b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqr
t(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/c^(5/2)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ax^2 + bx^3 + cx^4}\,dx = \int \sqrt{c\,x^4 + b\,x^3 + a\,x^2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.32 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$

3.32.1 Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx=\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{(b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-1/8*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*
x^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/4*(2*c*x+b)*(c*x^4+b*
x^3+a*x^2)^(1/2)/c/x
```

3.32.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx=\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{c}(b+2cx)+\frac{(b^2-4ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{\sqrt{a+x(b+cx)}}\right)}{4c^{3/2}x}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[c]*(b + 2*c*x) + ((b^2 - 4*a*c)*ArcTanh
[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])])/Sqrt[a + x*(b + c*x)]))/(
4*c^(3/2)*x)
```

3.32.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1965, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}dx$$

$$\Big\downarrow \text{1965}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{\left(b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

output
```
((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[
a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

3.32.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

3.32.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{4\sqrt{c\,x^{2}+bx+a}\,c^{\frac{3}{2}}x+4\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)ac-\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b^{2}+2b\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}{8c^{\frac{3}{2}}}$
risch	$\frac{(2cx+b)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{4cx}+\frac{(4ac-b^{2})\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{8c^{\frac{3}{2}}x\sqrt{c\,x^{2}+bx+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(4c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,x+2c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b+4\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)a\,c^{2}-\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right.\right.}{8c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,x}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^(3/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x+4*ln(2*(c*x^2+b*x+a)^(1/2)*c^
(1/2)+2*c*x+b)*a*c-ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^2+2*b*(c*x^
2+b*x+a)^(1/2)*c^(1/2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \left[-\frac{(b^2-4\,ac)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x+bc)}{16\,c^2x},\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*
arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c
*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]
```

3.32.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)
```

3.32.7 Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x, x)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{1}{8}\left(2\sqrt{cx^2+bx+a}\left(2x+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}\right)\operatorname{sgn}(x) - \frac{\left(b^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-4ac\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}\right)\operatorname{sgn}(x)}{8c^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(
abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sq
rt(a)*b*sqrt(c))*sgn(x)/c^(3/2)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x, x)
```

3.33 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} \, dx$

3.33.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} \, dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{a}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a
)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c
*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(
c*x^4+b*x^3+a*x^2)^(1/2)/x
```

3.33.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} \, dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)} + 4\sqrt{a}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) - b\log\left(b+2cx-2\sqrt{c}\sqrt{a+\right.\right.}{2\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] + 4*Sqrt[a]*Sqrt
[c]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - b*Log[b + 2*c*x
 - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x)
)])
```

3.33.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1968, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}dx$$

$$\Big\downarrow \text{1968}$$

$$\frac{1}{2}\int \frac{2a+bx}{\sqrt{cx^4+bx^3+ax^2}}dx+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{1980}$$

$$\frac{x\sqrt{a+bx+cx^2}\int \frac{2a+bx}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{1269}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(b\int \frac{1}{\sqrt{cx^2+bx+a}}dx+2a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{1092}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2b\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2a\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{1154}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-4a\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

output
```
Sqrt[a*x^2 + b*x^3 + c*x^4]/x + (x*Sqrt[a + b*x + c*x^2]*(-2*Sqrt[a]*ArcTa
nh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] + (b*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]))/(2*Sqrt[a*x^2 + b*x^3 + c*x
^4])
```

3.33.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1968
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Simp[(n - q)*(p/(m + p*(2*n - q) + 1))   Int[x^(m + q)*
(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; Free
Q[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
 + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

3.33.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$\frac{-2\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)\right)\sqrt{c}\,\sqrt{a}+\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}{2\sqrt{c}}$	95
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(2\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{c}-b\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)-2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}\right)}{2x\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}$	126

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c^(1/2)*(-2*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/
2)))*c^(1/2)*a^(1/2)+ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b+2*(c*x^2+
b*x+a)^(1/2)*c^(1/2))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.69

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$
$$= \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+2\,\sqrt{a}cx\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3}}{x^3}\right.}{4\,cx}\right.$$
$$-\frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-\sqrt{a}cx\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-}{2\,cx}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(8*a*b*
x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2
*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-
c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3
+ b*c*x^2 + a*c*x)) - sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*
a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c
*x^4 + b*x^3 + a*x^2)*c)/(c*x), 1/4*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4
+ b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*sqr
t(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
+ b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x
), 1/2*(2*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sq
rt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x)]
```

3.33.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx=\int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)
```

3.33.7 Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)
```

3.33.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2, x)
```

3.34 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$

3.34.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx=-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-1/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^
(1/2)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c
*x^2+b*x+a)^(1/2))*c^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(
c*x^4+b*x^3+a*x^2)^(1/2)/x^2
```

3.34.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$
$$=\frac{\sqrt{a+x(b+cx)}\left(bx\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\sqrt{a}\left(\sqrt{a+x(b+cx)}+\sqrt{c}x\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(\right.\right.\right.}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(b*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sq
rt[a]] - Sqrt[a]*(Sqrt[a + x*(b + c*x)] + Sqrt[c]*x*Log[b + 2*c*x - 2*Sqrt
[c]*Sqrt[a + x*(b + c*x)]])))/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

3.34.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1967, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{2}\int \frac{b+2cx}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{1980}$$

$$\frac{x\sqrt{a+bx+cx^2}\int \frac{b+2cx}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2c\int \frac{1}{\sqrt{cx^2+bx+a}}dx+b\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(b\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+4c\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(b\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-2b\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

output
```
-(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) + (x*Sqrt[a + b*x + c*x^2]*(-((b*ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + 2*Sqrt[c]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]))/(2*Sqrt[a*x^2 + b*x^
3 + c*x^4])
```

3.34.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

3.34.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{2\sqrt{c}\,\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)x\sqrt{a}+bx\ln\left(2\right)-bx\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)-2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{2x\sqrt{a}}$
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{x^{2}}+\frac{\left(\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)-\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{2\sqrt{a}}\right)\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{x\sqrt{c\,x^{2}+bx+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(2x^{2}\sqrt{c\,x^{2}+bx+a}\,c^{\frac{5}{2}}-c^{\frac{3}{2}}\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)bx-2\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}c^{\frac{3}{2}}+2c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,bx-}{2x^{2}\sqrt{c\,x^{2}+bx+a}\,a\,c^{\frac{3}{2}}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c^(1/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*x*a^(1/2)+b*x*ln(
2)-b*x*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))-2*a^(1/2)*(c*
x^2+b*x+a)^(1/2))/x/a^(1/2)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 653, normalized size of antiderivative = 3.77

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$

$$= \left[\frac{2\,a\sqrt{c}x^2\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4}}{x^3}\right.}{4\,ax^2}\right.$$

$$-\frac{4\,a\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right.}{4\,ax^2}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt(a)*b*x^2*log(-(8*
a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x
 + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), -1/4*(4*
a*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)
/(c^2*x^3 + b*c*x^2 + a*c*x)) - sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a
*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3
) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1
/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a
^2*x)) + a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3
+ a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3
+ a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*sqrt(-c)*x^2
*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*
c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2)]
```

3.34.6 Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3}\,dx = \int \frac{\sqrt{x^2\,(a + bx + cx^2)}}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

3.34.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.34.8 Giac [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3}\,dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^3}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^3}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3, x)
```

3.35 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4} dx$

3.35.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx=-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}$$

output
```
1/8*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)
)/a^(3/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/x^3-1/4*b*(c*x^4+b*x^3+a*x^2)^(1/2
)/a/x^2
```

3.35.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = -\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}+(b^2-4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

output
```
-1/4*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x
)] + (b^2 - 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]
]))/(a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])
```

3.35.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1967, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{4}\int \frac{b+2cx}{x\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{4}\left(-\frac{\int \frac{b^2-4ac}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \text{1951}$$

$$\frac{1}{4}\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3 + (-((b*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(a*x^2)) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 +
b*x^3 + c*x^4])])/(2*a^(3/2)))/4
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.35.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{\sqrt{c x^2+bx+a}\,(bx+2a)}{4x^2 a}+\frac{(4ac-b^2)\left(\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c x^2+bx+a}}{x\sqrt{a}}\right)\right)}{8a^{\frac{3}{2}}}$
risch	$-\frac{(bx+2a)\sqrt{x^2(c x^2+bx+a)}}{4x^3 a}-\frac{(4ac-b^2)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c x^2+bx+a}}{x}\right)\sqrt{x^2(c x^2+bx+a)}}{8a^{\frac{3}{2}}\,x\sqrt{c x^2+bx+a}}$
default	$-\frac{\sqrt{c x^4+b x^3+a x^2}\left(4c a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c x^2+bx+a}}{x}\right)x^2+2c\sqrt{c x^2+bx+a}\,b x^3-4c\sqrt{c x^2+bx+a}\,a x^2-\sqrt{a}\ln\left(\frac{2a+bx+2\sqrt{\cdots}}{\cdots}\right)\cdots\right)}{8x^3\sqrt{c x^2+bx+a}\,a^2}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(b*x+2*a)/x^2/a+1/8*(4*a*c-b^2)*(ln(2)-ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(3/2)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$$

$$= \left[-\frac{(b^2-4\,ac)\sqrt{a}x^3\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}(abx+2\,a^2 \cdots}{16\,a^2x^3}\right.$$

$$\left.-\frac{(b^2-4\,ac)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}(abx+2\,a^2)}{8\,a^2x^3}\right]$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*
a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt
(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*
x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(
a^2*x^3)]
```

3.35.6 Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4}\, dx = \int \frac{\sqrt{x^2 (a + bx + cx^2)}}{x^4}\, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)
```

3.35.7 Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4}\, dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^4}\, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)
```

3.35.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^4}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4, x)
```

3.36 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$

3.36.1 Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx=-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}$$

output
```
-1/16*b*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(
1/2))/a^(5/2)-1/3*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4-1/12*b*(c*x^4+b*x^3+a*x^2)
^(1/2)/a/x^3+1/24*(-8*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^2
```

3.36.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(-8a^2+3b^2x^2-2ax(b+4cx))+3b(b^2-4ac)\,x^3\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{}}{}\right.\right.}{24a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-8*a^2 + 3*b^
2*x^2 - 2*a*x*(b + 4*c*x)) + 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(Sqrt[c]*x - Sq
rt[a + x*(b + c*x)])/Sqrt[a]]))/(24*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])
```

3.36.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1967, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{6}\int \frac{b+2cx}{x^2\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{6}\left(-\frac{\int \frac{3b^2+2cxb-8ac}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{\int \frac{3b^2+2cxb-8ac}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{6}\left(-\frac{-\frac{\int \frac{3b\left(b^2-4ac\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{-\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$\Big\downarrow$ 1951

$$\frac{1}{6}\left(-\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$\Big\downarrow$ 219

$$\frac{1}{6}\left(-\frac{\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4 + (-1/2*(b*Sqrt[a*x^2 + b*x^3 + c*x^4
])/(a*x^3) - (-(((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (
3*b*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 +
c*x^4])])/(2*a^(3/2)))/(4*a))/6
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.36.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{b\,x^{3}\left(ac-\frac{b^{2}}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)+\left(-\frac{x(4cx+b)a^{\frac{3}{2}}}{3}+\frac{\sqrt{a}\,b^{2}x^{2}}{2}-\frac{4a^{\frac{5}{2}}}{3}\right)\sqrt{c\,x^{2}+bx+a}-\ln(2)x^{3}b\left(ac-\frac{b^{2}}{4}\right)}{4a^{\frac{5}{2}}\,x^{3}}$
risch	$-\frac{\left(8ac\,x^{2}-3b^{2}x^{2}+2abx+8a^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{24x^{4}a^{2}}+\frac{\left(4ac-b^{2}\right)b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{16a^{\frac{5}{2}}\,x\sqrt{c\,x^{2}+bx+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(12c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)b\,x^{3}+6c\sqrt{c\,x^{2}+bx+a}\,b^{2}x^{4}-12c\sqrt{c\,x^{2}+bx+a}\,ab\,x^{3}-3\sqrt{a}\,\ln\left(\frac{2a+bx}{}\right.\right.}{48x^{4}\sqrt{c\,x^{2}+bx}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x^3*(a*c-1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1
/2))+(-1/3*x*(4*c*x+b)*a^(3/2)+1/2*a^(1/2)*b^2*x^2-4/3*a^(5/2))*(c*x^2+b*x
+a)^(1/2)-ln(2)*x^3*b*(a*c-1/4*b^2))/a^(5/2)/x^3
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$
$$= \left[-\frac{3\,(b^3-4\,abc)\sqrt{a}x^4\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}(2\,a^2bx\right.}{96\,a^3x^4}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3
+ 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sq
rt(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(
a^3*x^4), 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x
^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x
^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^
4)]
```

3.36.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx = \int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x^5}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

3.36.7 Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^5} \, dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^5} \, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)
```

3.36.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^5} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^5} \, dx = \int \frac{\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}{x^5} \, dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5, x)
```

3.37 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6} dx$

3.37.1 Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6} dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2} + \frac{(b^2-4ac)(5b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}$$

output
```
1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b
*x^3+a*x^2)^(1/2))/a^(7/2)-1/4*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5-1/24*b*(c*x^4
+b*x^3+a*x^2)^(1/2)/a/x^4+1/96*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a
^2/x^3-1/192*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/x^2
```

3.37.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx =$$
$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(48a^3+15b^3x^3+8a^2x(b+3cx)-2abx^2(5b+26cx))+3(5b^4\right.}{192a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

output
```
-1/192*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3
 + 15*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]]))/(a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])
```

3.37.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1967, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{8}\int \frac{b+2cx}{x^3\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{8}\left(-\frac{\int \frac{5b^2+4cxb-12ac}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(-\frac{\int \frac{5b^2+4cxb-12ac}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{8}\left(-\frac{-\frac{\int \frac{b(15b^2-52ac)+2c(5b^2-12ac)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(-\frac{-\frac{\int \frac{b(15b^2-52ac)+2c(5b^2-12ac)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{8}\left(-\frac{-\frac{-\frac{\int \frac{3(b^2-4ac)(5b^2-4ac)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(-\frac{-\frac{-\frac{3(b^2-4ac)(5b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{1951}$$

$$\frac{1}{8}\left(-\frac{-\frac{\frac{3(b^2-4ac)(5b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

3.37. $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\left(-\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx}}{3ax^4}\right.$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

output
```
-1/4*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5 + (-1/3*(b*Sqrt[a*x^2 + b*x^3 + c*x^4
])/(a*x^4) - (-1/2*((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3)
- (-((b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (3*(b^2
- 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b
*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))/(6*a))/8
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{x^4\left(ac-\frac{5b^2}{4}\right)\left(ac-\frac{b^2}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)+\left(\frac{5\left(\frac{26cx}{5}+b\right)x^2 b\,a^{\frac{3}{2}}}{12}+\left(-c\,x^2-\frac{1}{3}bx\right)a^{\frac{5}{2}}-\frac{5\sqrt{a}\,b^3x^3}{8}-2a^{\frac{7}{2}}\right)\sqrt{c\,x^2+bx+}}{8a^{\frac{7}{2}}x^4}$
risch	$-\frac{\left(-52abc\,x^3+15b^3x^3+24a^2c\,x^2-10a\,b^2x^2+8a^2bx+48a^3\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{192x^5a^3}+\frac{\left(16a^2c^2-24a\,b^2c+5b^4\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c}}{x}\right.}{128a^{\frac{7}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(48c^2a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^4+24c^2\sqrt{c\,x^2+bx+a}\,ab\,x^5-72c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(7/2)*(x^4*(a*c-5/4*b^2)*(a*c-1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x/a^(1/2))+(5/12*(26/5*c*x+b)*x^2*b*a^(3/2)+(-c*x^2-1/3*b*x)
*a^(5/2)-5/8*a^(1/2)*b^3*x^3-2*a^(7/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^4*(a*c
-5/4*b^2)*(a*c-1/4*b^2))/x^4
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$$
$$= \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,(8\,a^3bx+48\,a}{768\,a^4x^5}\right.$$
$$-\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,(8\,a^3bx+48\,a^4+(15\,ab^3-52\,a}{384\,a^4x^5}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(a))/x^3) - 4*(8*a^3*b*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a
^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*
(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x
^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^3*b
*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*
sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5)]
```

3.37.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = \int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x^6}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

3.37.7 Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx=\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)
```

3.37.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^6}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6, x)
```

3.38 $\int x(ax^2+bx^3+cx^4)^{3/2}\,dx$

3.38.1 Optimal result

Integrand size = 22, antiderivative size = 422

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\frac{(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b(3465b^6-30660ab^4c+81648a^2b^2c^2-58816a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x}-\frac{b(231b^4-1560ab^2c+2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{(99b^4-568ab^2c+560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{x^3(b(11b^2+68ac)+10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2}+\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}+\frac{3(b^2-4ac)^2\,(33b^4-72ab^2c+16a^2c^2)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
1/112*x*(14*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c+3/32768*(-4*a*c+b^2)^2*(1
6*a^2*c^2-72*a*b^2*c+33*b^4)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)
^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(13/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/286720*(-
6720*a^3*c^3+18896*a^2*b^2*c^2-8988*a*b^4*c+1155*b^6)*(c*x^4+b*x^3+a*x^2)^
(1/2)/c^5-1/573440*b*(-58816*a^3*c^3+81648*a^2*b^2*c^2-30660*a*b^4*c+3465*
b^6)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^6/x-1/71680*b*(2416*a^2*c^2-1560*a*b^2*c+
231*b^4)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(560*a^2*c^2-568*a*b^2*c+
99*b^4)*x^2*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/4480*x^3*(b*(68*a*c+11*b^2)+10
*c*(-28*a*c+11*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2
```

3.38.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.72

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\frac{x\sqrt{a+x(b+cx)}\Big(2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^7+2310b^6cx+84b^5c(365a-22cx^2)+24b^4c^2\cdots}{\cdots}$$

input
```
Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^7 + 231
0*b^6*c*x + 84*b^5*c*(365*a - 22*c*x^2) + 24*b^4*c^2*x*(-749*a + 66*c*x^2)
 + 32*b^2*c^3*x*(1181*a^2 - 284*a*c*x^2 + 40*c^2*x^4) - 16*b^3*c^2*(5103*a
^2 - 780*a*c*x^2 + 88*c^2*x^4) + 4480*c^4*x*(-3*a^3 + 2*a^2*c*x^2 + 24*a*c
^2*x^4 + 16*c^3*x^6) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^2 + 104*a*c^2*x^4 +
 1360*c^3*x^6)) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(1146880*c^(13/2)*Sqrt[x
^2*(a + x*(b + c*x))])
```

3.38.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.682$, Rules used = {1966, 27, 1992, 27, 1996, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.38. $\int x(ax^2+bx^3+cx^4)^{3/2}\,dx$

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}dx$$

$\Big\downarrow$ 1966

$$\frac{3\int -\frac{1}{2}x^2\left(8ab+\left(11b^2-28ac\right)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{112c}+\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}$$

$\Big\downarrow$ 27

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\int x^2\left(8ab+\left(11b^2-28ac\right)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{224c}$$

$\Big\downarrow$ 1992

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{\int -\frac{x^4\left(8ab\left(11b^2-52ac\right)+\left(99b^4-568acb^2+560a^2c^2\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{60c}+\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}\right)}{224c}$$

$\Big\downarrow$ 27

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\int \frac{x^4\left(8ab\left(11b^2-52ac\right)+\left(99b^4-568acb^2+560a^2c^2\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{120c}\right)}{224c}$$

$\Big\downarrow$ 1996

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{\int \frac{3x^3\left(2a\left(99b^4-568acb^2+560a^2c^2\right)+b\left(23\cdots\right)\right)}{2\sqrt{cx^4+bx^3+\cdots}}}{4c}}{120c}\cdots\right)}{224c}$$

$\Big\downarrow$ 27

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\int \frac{x^3\left(2a\left(99b^4-568acb^2+560a^2c^2\right)+b\left(23\cdots\right)\right)}{\sqrt{cx^4+bx^3+\cdots}}}{8c}}{120c}\cdots\right)}{224c}$$

$\Big\downarrow$ 1996

3.38. $\int x\left(ax^2+bx^3+cx^4\right)^{3/2}dx$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+}}{3c}\right.}{}}{}\right.}{224c}$$

$\downarrow$ 27

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+}}{3c}\right.}{}}{}\right.}{224c}$$

$\downarrow$ 1996

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+}}{3c}\right.}{}}{}\right.}{}$$

$\downarrow$ 27

3.38. $\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+}}{3c}\right.\right.$$

$\downarrow$ 1996

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+}}{3c}\right.\right.$$

$\downarrow$ 27

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+}}{3c}\right.\right.$$

$\downarrow$ 1961

3.38. $\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+\cdots}}{3c}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 1092

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+\cdots}}{3c}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 219

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+\cdots}}{3c}\right.}{\cdots}}{\cdots}\right.$$

3.38. $\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

input
```
Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(112*c) - (3*((x^3*(b*(11
*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(6
0*c) - (((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(4*c) - (3*((b*(231*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(3*c) - (((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 -
6720*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c) - ((b*(3465*b^6 - 30660*a
*b^4*c + 81648*a^2*b^2*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
c*x) - (105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a +
b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c))/(6*c)))/(8*c))/(120*c)))/(224*c
)
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.38. $\int x(ax^2+bx^3+cx^4)^{3/2}\,dx$

3.38.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.78

method	result
risch	$\frac{(71680c^7x^7+87040b\,c^6x^6+107520a\,c^6x^5+1280b^2c^5x^5+6656ab\,c^5x^4-1408b^3c^4x^4+8960a^2c^5x^3-9088a\,b^2c^4x^3+1584b^4c^3x^3-19328a\ldots}{}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(-80640c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}a\,b^2x-127680c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2b^2x+85680c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,a\,b^4x+143360x^3(c\,x^2+b\ldots\right.}{}$

input
```
int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/573440*(71680*c^7*x^7+87040*b*c^6*x^6+107520*a*c^6*x^5+1280*b^2*c^5*x^5+
6656*a*b*c^5*x^4-1408*b^3*c^4*x^4+8960*a^2*c^5*x^3-9088*a*b^2*c^4*x^3+1584
*b^4*c^3*x^3-19328*a^2*b*c^4*x^2+12480*a*b^3*c^3*x^2-1848*b^5*c^2*x^2-1344
0*a^3*c^4*x+37792*a^2*b^2*c^3*x-17976*a*b^4*c^2*x+2310*b^6*c*x+58816*a^3*b
*c^3-81648*a^2*b^3*c^2+30660*a*b^5*c-3465*b^7)/c^6*(x^2*(c*x^2+b*x+a))^(1/
2)/x+3/32768*(256*a^4*c^4-1280*a^3*b^2*c^3+1120*a^2*b^4*c^2-336*a*b^6*c+33
*b^8)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*(x^2*(c*x^2+b*x
+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.57

$$\int x(ax^2+bx^3+cx^4)^{3/2}\,dx = \left[\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+b\ldots}}{}\right.}{}\right.$$

$$-\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(\ldots}{}$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^
3 + 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7
+ 87040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 588
16*a^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^
6)*x^4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^
3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7
*x), -1/1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b
^2*c^3 + 256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040
*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*
b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4
+ 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 156
0*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 188
96*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]
```

3.38.6 Sympy [F]

$$\int x(ax^2 + bx^3 + cx^4)^{3/2}\, dx = \int x\left(x^2(a + bx + cx^2)\right)^{\frac{3}{2}}\, dx$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.38.7 Maxima [F]

$$\int x(ax^2 + bx^3 + cx^4)^{3/2}\, dx = \int (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x\, dx$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.21

$$\int x(ax^2+bx^3+cx^4)^{3/2}\,dx = \frac{1}{573440}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\,cx\mathrm{sgn}(x)+17\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^6\mathrm{sgn}(x)+84\,a\right.\right.\right.\right.\right.\right.}{c^7}$$
$$-\frac{3\left(33\,b^8\mathrm{sgn}(x)-336\,ab^6c\mathrm{sgn}(x)+1120\,a^2b^4c^2\mathrm{sgn}(x)-1280\,a^3b^2c^3\mathrm{sgn}(x)+256\,a^4c^4\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x\right.\right.\right.}{32768\,c^{\frac{13}{2}}}$$
$$+\frac{\left(3465\,b^8\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-35280\,ab^6c\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+117600\,a^2b^4c^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-13440\right.}{}$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*x*sgn(x) + 17*b*sg
n(x))*x + (b^2*c^6*sgn(x) + 84*a*c^7*sgn(x))/c^7)*x - (11*b^3*c^5*sgn(x) -
 52*a*b*c^6*sgn(x))/c^7)*x + (99*b^4*c^4*sgn(x) - 568*a*b^2*c^5*sgn(x) + 5
60*a^2*c^6*sgn(x))/c^7)*x - (231*b^5*c^3*sgn(x) - 1560*a*b^3*c^4*sgn(x) +
2416*a^2*b*c^5*sgn(x))/c^7)*x + (1155*b^6*c^2*sgn(x) - 8988*a*b^4*c^3*sgn(
x) + 18896*a^2*b^2*c^4*sgn(x) - 6720*a^3*c^5*sgn(x))/c^7)*x - (3465*b^7*c*
sgn(x) - 30660*a*b^5*c^2*sgn(x) + 81648*a^2*b^3*c^3*sgn(x) - 58816*a^3*b*c
^4*sgn(x))/c^7) - 3/32768*(33*b^8*sgn(x) - 336*a*b^6*c*sgn(x) + 1120*a^2*b
^4*c^2*sgn(x) - 1280*a^3*b^2*c^3*sgn(x) + 256*a^4*c^4*sgn(x))*log(abs(2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(13/2) + 1/1146880*(3465
*b^8*log(abs(b - 2*sqrt(a)*sqrt(c))) - 35280*a*b^6*c*log(abs(b - 2*sqrt(a)
*sqrt(c))) + 117600*a^2*b^4*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 134400*a
^3*b^2*c^3*log(abs(b - 2*sqrt(a)*sqrt(c))) + 26880*a^4*c^4*log(abs(b - 2*s
qrt(a)*sqrt(c))) + 6930*sqrt(a)*b^7*sqrt(c) - 61320*a^(3/2)*b^5*c^(3/2) +
163296*a^(5/2)*b^3*c^(5/2) - 117632*a^(7/2)*b*c^(7/2))*sgn(x)/c^(13/2)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int x(ax^2+bx^3+cx^4)^{3/2}\,dx = \int x\,\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.38. $\int x(ax^2 + bx^3 + cx^4)^{3/2}\, dx$

3.39 $\int (ax^2+bx^3+cx^4)^{3/2}\,dx$

3.39.1 Optimal result

Integrand size = 20, antiderivative size = 364

$$\begin{aligned}\int (ax^2+bx^3+cx^4)^{3/2}\,dx &= -\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}\\ &+\frac{(315b^6-2520ab^4c+5488a^2b^2c^2-2048a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x}\\ &+\frac{(7b^2-32ac)(3b^2-4ac)x\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{b(9b^2-44ac)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}\\ &+\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x(ax^2+bx^3+cx^4)^{3/2}\\ &-\frac{3b(b^2-4ac)^2(3b^2-4ac)x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

output
```
1/7*x*(c*x^4+b*x^3+a*x^2)^(3/2)-3/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*x*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(1
1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/17920*b*(1168*a^2*c^2-728*a*b^2*c+105*b^4
)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(-2048*a^3*c^3+5488*a^2*b^2*c^2-25
20*a*b^4*c+315*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^5/x+1/4480*(-32*a*c+7*b^2)
*(-4*a*c+3*b^2)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/2240*b*(-44*a*c+9*b^2)*x
^2*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/280*x^3*(10*b*c*x+24*a*c+b^2)*(c*x^4+b*
x^3+a*x^2)^(1/2)/c
```

3.39.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.66

$$\int (ax^2 + bx^3 + cx^4)^{3/2} \, dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(315b^6 - 210b^5cx + 16b^3c^2x(91a - 9cx^2) + 168b^4c(-\right.\right.}{}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^6 - 210*b
^5*c*x + 16*b^3*c^2*x*(91*a - 9*c*x^2) + 168*b^4*c*(-15*a + c*x^2) + 1024*
c^3*(a + c*x^2)^2*(-2*a + 5*c*x^2) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^2 + 8*
c^2*x^4) + 32*b*c^3*x*(-73*a^2 + 22*a*c*x^2 + 200*c^2*x^4)) + 105*b*(b^2 -
 4*a*c)^2*(3*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
))/(71680*c^(11/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.39.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1953, 1992, 27, 1996, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2 + bx^3 + cx^4)^{3/2} \, dx$$

$$\Big\downarrow \text{1953}$$

$$\frac{3}{14}\int x^2(2a+bx)\sqrt{cx^4+bx^3+ax^2}dx + \frac{1}{7}x(ax^2+bx^3+cx^4)^{3/2}$$

$$\Big\downarrow \text{1992}$$

$$\frac{3}{14}\left(\frac{\int -\frac{x^4(8a(b^2-6ac)+b(9b^2-44ac)x)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{60c} + \frac{x^3(24ac+b^2+10bcx)\sqrt{ax^2+bx^3+cx^4}}{60c}\right) + \frac{1}{7}x(ax^2+bx^3+cx^4)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\int \frac{x^4\left(8a\left(b^2-6ac\right)+b\left(9b^2-44ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{120c}\right)+$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 1996

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{\int \frac{3x^3\left(2ab\left(9b^2-44ac\right)+\left(7b^2-32ac\right)\left(3b^2-4ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}}{4c}}{120c}\right.$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 27

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\int \frac{x^3\left(2ab\left(9b^2-44ac\right)+\left(7b^2-32ac\right)\left(3b^2-4ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}}{8c}}{120c}\right.$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 1996

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int}{}\right.}{}}{120c}\right.$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 27

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int}{}\right.}{}}{120c}\right.$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

3.39. $\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

$$\downarrow 1996$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b\left(\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\downarrow 27$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b\left(\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\downarrow 1996$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b\left(\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\downarrow 27$$

3.39. $\int \left(ax^2+bx^3+cx^4\right)^{3/2}\, dx$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b(\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 1961

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b(\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 1092

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b(\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 219

3.39. $\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

$$\frac{3}{14}\left(\frac{x^3(24ac+b^2+10bcx)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2(9b^2-44ac)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x(7b^2-32ac)(3b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b(\cdots}{\cdots}\right)}{\cdots}}{\cdots}\right)$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 + (3*((x^3*(b^2 + 24*a*c + 10*b*c*x)*S
qrt[a*x^2 + b*x^3 + c*x^4])/(60*c) - ((b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(4*c) - (3*(((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*x*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(3*c) - ((b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqr
t[a*x^2 + b*x^3 + c*x^4])/(2*c) - (((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2
*c^2 - 2048*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*x) - (105*b*(b^2 - 4*
a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c
))/(6*c)))/(8*c))/(120*c)))/14
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1953
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[x*((a*x^q + b*x^n + c*x^(2*n - q))^p/(p*(2*n - q) + 1)), x] + Sim
p[(n - q)*(p/(p*(2*n - q) + 1))   Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^
n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2
*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
 && NeQ[p*(2*n - q) + 1, 0]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.39. $\int (ax^2 + bx^3 + cx^4)^{3/2}\, dx$

3.39.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-5120c^6x^6-6400b\,c^5x^5-8192a\,c^5x^4-128b^2c^4x^4-704ab\,c^4x^3+144b^3c^3x^3-1024a^2c^4x^2+992a\,b^2c^3x^2-168b^4c^2x^2+2336a^2b\,c^3x-}{35840c^5x}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(10240x^2(c\,x^2+bx+a)^{\frac{5}{2}}c^{\frac{11}{2}}-7680c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{5}{2}}bx-4096c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{5}{2}}a+4480c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}abx+6\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35840*(-5120*c^6*x^6-6400*b*c^5*x^5-8192*a*c^5*x^4-128*b^2*c^4*x^4-704*
a*b*c^4*x^3+144*b^3*c^3*x^3-1024*a^2*c^4*x^2+992*a*b^2*c^3*x^2-168*b^4*c^2
*x^2+2336*a^2*b*c^3*x-1456*a*b^3*c^2*x+210*b^5*c*x+2048*a^3*c^3-5488*a^2*b
^2*c^2+2520*a*b^4*c-315*b^6)/c^5*(x^2*(c*x^2+b*x+a))^(1/2)/x+3/2048*b*(64*
a^3*c^3-80*a^2*b^2*c^2+28*a*b^4*c-3*b^6)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*(x^2*(c*x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.53

$$\int (ax^2+bx^3+cx^4)^{3/2}\,dx = \left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b}{x}\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(
c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*
b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 +
64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4
)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*
c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(5120*c^7
*x^6 + 6400*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 20
48*a^3*c^4 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^
3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 72
8*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]
```

3.39.6 Sympy [F]

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int \left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)
```

3.39.7 Maxima [F]

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.15

$$\int (ax^2+bx^3+cx^4)^{3/2}\,dx = \frac{1}{35840}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\Bigg(2\Bigg(8\Bigg(10\,(4\,cx\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x))x+\frac{b^2c^5\mathrm{sgn}(x)+64\,ac^6\mathrm{sgn}(x}{c^6}$$

$$+\frac{3\,(3\,b^7\mathrm{sgn}(x)-28\,ab^5c\mathrm{sgn}(x)+80\,a^2b^3c^2\mathrm{sgn}(x)-64\,a^3bc^3\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2048\,c^{\frac{11}{2}}}$$

$$-\frac{\left(315\,b^7\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-2940\,ab^5c\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+8400\,a^2b^3c^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-6720\,a^3bc^3}{71680\,c^{\frac{11}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sgn(x) + 5*b*sgn(x))*
x + (b^2*c^5*sgn(x) + 64*a*c^6*sgn(x))/c^6)*x - (9*b^3*c^4*sgn(x) - 44*a*b
*c^5*sgn(x))/c^6)*x + (21*b^4*c^3*sgn(x) - 124*a*b^2*c^4*sgn(x) + 128*a^2*
c^5*sgn(x))/c^6)*x - (105*b^5*c^2*sgn(x) - 728*a*b^3*c^3*sgn(x) + 1168*a^2
*b*c^4*sgn(x))/c^6)*x + (315*b^6*c*sgn(x) - 2520*a*b^4*c^2*sgn(x) + 5488*a
^2*b^2*c^3*sgn(x) - 2048*a^3*c^4*sgn(x))/c^6) + 3/2048*(3*b^7*sgn(x) - 28*
a*b^5*c*sgn(x) + 80*a^2*b^3*c^2*sgn(x) - 64*a^3*b*c^3*sgn(x))*log(abs(2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2) - 1/71680*(315*b^
7*log(abs(b - 2*sqrt(a)*sqrt(c))) - 2940*a*b^5*c*log(abs(b - 2*sqrt(a)*sqr
t(c))) + 8400*a^2*b^3*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 6720*a^3*b*c^3
*log(abs(b - 2*sqrt(a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*
b^4*c^(3/2) + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sgn(x)/c^(
11/2)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int (ax^2+bx^3+cx^4)^{3/2}\,dx = \int (c\,x^4+b\,x^3+a\,x^2)^{3/2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.40 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx$

3.40.1 Optimal result

Integrand size = 24, antiderivative size = 288

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx &= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} \\ &- \frac{b(105b^4-760ab^2c+1296a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} \\ &- \frac{x(b(7b^2+12ac)+6c(7b^2-20ac)x)\sqrt{ax^2+bx^3+cx^4}}{960c^2} \\ &+ \frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx} \\ &+ \frac{(b^2-4ac)^2(7b^2-4ac)x\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

output
```
1/60*(10*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x+1/1024*(-4*a*c+b^2)^2*(-4*
a*c+7*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x
+a)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/3840*(240*a^2*c^2-216*a*b^2*
c+35*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/7680*b*(1296*a^2*c^2-760*a*b^2*c
+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/960*x*(b*(12*a*c+7*b^2)+6*c*(-
20*a*c+7*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2
```

3.40.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.73

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5+70b^4cx+8b^3c(95a-7cx^2)\right.}{}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5 + 70*b
^4*c*x + 8*b^3*c*(95*a - 7*c*x^2) + 48*b^2*c^2*x*(-9*a + c*x^2) + 160*c^3*
x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4) + 16*b*c^2*(-81*a^2 + 18*a*c*x^2 + 104*
c^2*x^4)) - 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]]))/(15360*c^(9/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.40.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1966, 27, 1981, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{1966}$$

$$\frac{\int -\frac{1}{2}\left(4ab+\left(7b^2-20ac\right)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{20c}+\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\int \left(4ab+\left(7b^2-20ac\right)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{40c}$$

$$\Big\downarrow \text{1981}$$

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{\int -\frac{x^2\left(4ab\left(7b^2-36ac\right)+\left(35b^4-216acb^2+240a^2c^2\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{24c}+\frac{x\sqrt{ax^2+bx^3+cx^4}\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)}{24c}}{40c}$$

$\downarrow$ 27

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\int \frac{x^2\left(4ab\left(7b^2-36ac\right)+\left(35b^4-216acb^2+240a^2c^2\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{48c}}{40c}$$

$\downarrow$ 1996

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(35b^4-216acb^2+240a^2c^2\right)+b\left(105b^4-760acb^2+\cdots\right)\right)}{2\sqrt{cx^4+bx^3+ax^2}}\cdots}{2c}}{48c}}{40c}$$

$\downarrow$ 27

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(35b^4-216acb^2+240a^2c^2\right)+b\left(105b^4-760acb^2+\cdots\right)\right)}{\sqrt{cx^4+bx^3+ax^2}}\cdots}{4c}}{48c}}{40c}$$

$\downarrow$ 1996

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\int \frac{15\left(\cdots\right)}{\cdots}}{4c}}{48c}}{40c}$$

$\downarrow$ 27

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15\left(b^2\cdots\right)}{\cdots}}{4c}}{48c}}{40c}$$

$\downarrow$ 1961

3.40. $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx$

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15x\left(b\cdots\right.}{4c}}{48c}\cdots}{40c}$$

$\downarrow$ 1092

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15x\left(b\cdots\right.}{\cdots}}{48c}\cdots}{40c}$$

$\downarrow$ 219

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15x\left(b\cdots\right.}{\cdots}}{48c}\cdots}{40c}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

output
```
((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) - ((x*(b*(7*b^2 +
12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c) - ((
(35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c) -
((b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c
*x) - (15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*
x^3 + c*x^4]))/(4*c))/(48*c))/(40*c)
```

3.40. $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx$

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

3.40. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx$

rule 1981
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_)*((A_) + (
B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*(
2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n
 - q)*(p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)))   Int[x^q*(2*
a*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n -
q) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1)
)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b
, c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] &
& NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q +
(n - q)*(2*p + 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.40.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(-1280c^5x^5-1664b\,c^4x^4-2240a\,c^4x^3-48b^2c^3x^3-288ab\,c^3x^2+56c^2x^2b^3-480a^2c^3x+432a\,b^2c^2x-70xc\,b^4+1296a^2b\,c^2-760a\,b^3c+}{7680c^4x}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}(2560x(c\,x^2+bx+a)^{\frac{5}{2}}c^{\frac{9}{2}}-640c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}ax-960c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2x-1792c^{\frac{7}{2}}(c\,x^2+bx+a)^{\frac{5}{2}}b+1120c^{\frac{7}{2}}}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(-1280*c^5*x^5-1664*b*c^4*x^4-2240*a*c^4*x^3-48*b^2*c^3*x^3-288*a*
b*c^3*x^2+56*b^3*c^2*x^2-480*a^2*c^3*x+432*a*b^2*c^2*x-70*b^4*c*x+1296*a^2
*b*c^2-760*a*b^3*c+105*b^5)/c^4*(x^2*(c*x^2+b*x+a))^(1/2)/x-1/1024*(64*a^3
*c^3-144*a^2*b^2*c^2+60*a*b^4*c-7*b^6)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*(x^2*(c*x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.65

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \Bigg[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+}}{x}\right.}{}$$

$$-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(1280\,c^6x^5+1664\,bc^5}{}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*
x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5
*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(
7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)
*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c
 + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(1280*c^6*x
^5 + 1664*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b
^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 -
 216*a*b^2*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

3.40.6 Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} \, dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x} \, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)
```

3.40.7 Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} \, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x} \, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.23

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} \, dx = \frac{1}{7680} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(2 \left(8 \left(10 \, cx\text{sgn}(x) + 13 \, b\text{sgn}(x)\right)x + \frac{3 \, b^2 c^4 \text{sgn}(x)}{}\right.\right.\right.\right.$$
$$- \frac{(7 \, b^6 \text{sgn}(x) - 60 \, ab^4 c\text{sgn}(x) + 144 \, a^2 b^2 c^2 \text{sgn}(x) - 64 \, a^3 c^3 \text{sgn}(x)) \log \left(\left|2 \left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{1024 \, c^{\frac{9}{2}}}$$
$$+ \frac{\left(105 \, b^6 \log \left(\left|b - 2 \sqrt{a}\sqrt{c}\right|\right) - 900 \, ab^4 c \log \left(\left|b - 2 \sqrt{a}\sqrt{c}\right|\right) + 2160 \, a^2 b^2 c^2 \log \left(\left|b - 2 \sqrt{a}\sqrt{c}\right|\right) - 960 \, a^3 c^3 \log\right.}{15360 \, c^{\frac{9}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sgn(x) + 13*b*sgn(x))*x +
 (3*b^2*c^4*sgn(x) + 140*a*c^5*sgn(x))/c^5)*x - (7*b^3*c^3*sgn(x) - 36*a*b
*c^4*sgn(x))/c^5)*x + (35*b^4*c^2*sgn(x) - 216*a*b^2*c^3*sgn(x) + 240*a^2*
c^4*sgn(x))/c^5)*x - (105*b^5*c*sgn(x) - 760*a*b^3*c^2*sgn(x) + 1296*a^2*b
*c^3*sgn(x))/c^5) - 1/1024*(7*b^6*sgn(x) - 60*a*b^4*c*sgn(x) + 144*a^2*b^2
*c^2*sgn(x) - 64*a^3*c^3*sgn(x))*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*sqrt(c) + b))/c^(9/2) + 1/15360*(105*b^6*log(abs(b - 2*sqrt(a)*sqrt(c
))) - 900*a*b^4*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2160*a^2*b^2*c^2*log(a
bs(b - 2*sqrt(a)*sqrt(c))) - 960*a^3*c^3*log(abs(b - 2*sqrt(a)*sqrt(c))) +
 210*sqrt(a)*b^5*sqrt(c) - 1520*a^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/
2))*sgn(x)/c^(9/2)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x}\,dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x, x)
```

3.41 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx$

3.41.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx &= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} \\ &- \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5} \\ &- \frac{3b(b^2-4ac)^2 x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

output
```
-1/16*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^3+a*x^2
)^(5/2)/c/x^5-3/256*b*(-4*a*c+b^2)^2*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^
2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/12
8*b*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/x
```

3.41.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(15b^4-10b^3cx+128c^2(a+cx^2)^2+4b\right.\right.}{1280c}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3
*c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a
+ 22*c*x^2)) + 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(
b + c*x)]]))/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.41.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1964, 1965, 1965, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1964}$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\int \frac{\left(cx^4+bx^3+ax^2\right)^{3/2}}{x^3}dx}{2c}$$

$$\Big\downarrow \text{1965}$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}dx}{16c}\right)}{2c}$$

$$\Big\downarrow \text{1965}$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{\left(b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}\right)}{2c}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

output
```
(a*x^2 + b*x^3 + c*x^4)^(5/2)/(5*c*x^5) - (b*(((b + 2*c*x)*(a*x^2 + b*x^3
+ c*x^4)^(3/2))/(8*c*x^3) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a*x^2 + b*
x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*Sqrt[a*x^2 + b*x^3
 + c*x^4])))/(16*c)))/(2*c)
```

3.41. $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}\,dx$

3.41.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1964
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Simp[b/(2*c)   Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c
*x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
 q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p
, q] && EqQ[m + p*(n - 1) - 1, 0]
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

3.41. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx$

3.41.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-\frac{15b\left(ac-\frac{b^2}{4}\right)^2\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{16}+\left(\left(\frac{1}{16}b^2x^2+\frac{7}{16}abx+a^2\right)c^{\frac{5}{2}}-\frac{25b^2\left(\frac{bx}{10}+a\right)c^{\frac{3}{2}}}{32}+\left(\frac{11}{8}b\,x^3+2a\,x^2\right)c^{\frac{7}{2}}+c^{\frac{9}{2}}x^4+\frac{15\sqrt{c}}{128}\right.}{5c^{\frac{7}{2}}}$
risch	$\frac{\left(128c^4x^4+176b\,c^3x^3+256a\,c^3x^2+8b^2c^2x^2+56ab\,c^2x-10b^3cx+128a^2c^2-100a\,b^2c+15b^4\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{640c^3x}-\frac{3b\left(16a^2c^2-\right.}{}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(256\left(c\,x^2+bx+a\right)^{\frac{5}{2}}c^{\frac{7}{2}}-160c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}bx-80c^{\frac{5}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}b^2-240c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,abx+60c\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5/c^(7/2)*(-15/16*b*(a*c-1/4*b^2)^2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c
*x+b)+((1/16*b^2*x^2+7/16*a*b*x+a^2)*c^(5/2)-25/32*b^2*(1/10*b*x+a)*c^(3/2
)+(11/8*b*x^3+2*a*x^2)*c^(7/2)+c^(9/2)*x^4+15/128*c^(1/2)*b^4)*(c*x^2+b*x+
a)^(1/2))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.94

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}\,dx=\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4}{x}\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8
*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*
c)*x)/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128
*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt
(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)
/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c
- 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2
- 28*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]
```

3.41.6 Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)
```

3.41.7 Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.39

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \frac{1}{640}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\Bigg(2\,(8\,cx\mathrm{sgn}(x)+11\,b\mathrm{sgn}(x))x+\frac{b^2c^3\mathrm{sgn}(x)+32\,ac}{c^4}$$
$$+\frac{3\,(b^5\mathrm{sgn}(x)-8\,ab^3c\mathrm{sgn}(x)+16\,a^2bc^2\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^5\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-120\,ab^3c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+240\,a^2bc^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^4\sqrt{c}-2\right.}{1280\,c^{\frac{7}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*sgn(x) + 11*b*sgn(x))*x + (b^2
*c^3*sgn(x) + 32*a*c^4*sgn(x))/c^4)*x - (5*b^3*c^2*sgn(x) - 28*a*b*c^3*sgn
(x))/c^4)*x + (15*b^4*c*sgn(x) - 100*a*b^2*c^2*sgn(x) + 128*a^2*c^3*sgn(x)
)/c^4) + 3/256*(b^5*sgn(x) - 8*a*b^3*c*sgn(x) + 16*a^2*b*c^2*sgn(x))*log(a
bs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2) - 1/1280*(1
5*b^5*log(abs(b - 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c*log(abs(b - 2*sqrt(a)*
sqrt(c))) + 240*a^2*b*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^4
*sqrt(c) - 200*a^(3/2)*b^2*c^(3/2) + 256*a^(5/2)*c^(5/2))*sgn(x)/c^(7/2)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^2}\,dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2, x)
```

3.42 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx$

3.42.1 Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = -\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} + \frac{3(b^2-4ac)^2\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
1/8*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x^3+3/128*(-4*a*c+b^2)^2*x*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/
(c*x^4+b*x^3+a*x^2)^(1/2)-3/64*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^
(1/2)/c^2/x
```

3.42.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \frac{(x^2(a+x(b+cx)))^{3/2}\left(\frac{\sqrt{c}(b+2cx)(-3b^2+8bcx+4c(5a+2cx^2))}{a+x(b+cx)} + \frac{3(b^2-4ac)^2\,\text{arctanh}\left(\frac{\quad}{-\sqrt{c}\ldots}\right)}{(a+x(b+cx))^{3\ldots}}\right.}{64c^{5/2}x^3}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

output
```
((x^2*(a + x*(b + c*x)))^(3/2)*((Sqrt[c]*(b + 2*c*x)*(-3*b^2 + 8*b*c*x + 4
*c*(5*a + 2*c*x^2)))/(a + x*(b + c*x)) + (3*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[
c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(a + x*(b + c*x))^(3/2)))/(64*c
^(5/2)*x^3)
```

3.42.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1965, 1965, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1965}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}dx}{16c}$$

$$\Big\downarrow \text{1965}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{\left(b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}\right)}{16c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} - \frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

output
```
((b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) - (3*(b^2 - 4*a*c)*(
((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[
a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])))/(16*c)
```

3.42.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

3.42.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{\frac{3\left(ac-\frac{b^2}{4}\right)^2\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{8}+\frac{5\sqrt{c\,x^2+bx+a}\,\left(b\left(\frac{bx}{10}+a\right)c^{\frac{3}{2}}+\left(\frac{6}{5}b\,x^2+2ax\right)c^{\frac{5}{2}}-\frac{3\sqrt{c}\,b^3}{20}+\frac{4c^{\frac{7}{2}}x^3}{5}\right)}{16}}{c^{\frac{5}{2}}}$
risch	$\frac{\left(16c^3x^3+24b\,c^2x^2+40a\,c^2x+2b^2cx+20abc-3b^3\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{64c^2x}+\frac{3\left(16a^2c^2-8a\,b^2c+b^4\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)\sqrt{x}}{128c^{\frac{5}{2}}x\sqrt{c\,x^2+bx+a}}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(32x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{7}{2}}+16c^{\frac{5}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}b+48c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,ax-12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,b^2x+24c^{\frac{5}{2}}\sqrt{c}\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3/8/c^(5/2)*((a*c-1/4*b^2)^2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)+5/6
*(c*x^2+b*x+a)^(1/2)*(b*(1/10*b*x+a)*c^(3/2)+(6/5*b*x^2+2*a*x)*c^(5/2)-3/2
0*c^(1/2)*b^3+4/5*c^(7/2)*x^3))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.94

$$\int\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx=\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac}{x}\right.}{}\right.$$

$$-\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\left(16\,c^4x^3+24\,bc^3x^2-3\,b^3c+20\,abc^2+\right.}{128\,c^3x}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c
*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x
)/x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 +
20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^
2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2
 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a
*x^2))/(c^3*x)]
```

3.42.6 Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^3}\,dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)
```

3.42.7 Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^3}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.36

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{64}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\,cx\mathrm{sgn}(x)+3\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^2\mathrm{sgn}(x)+20\,ac^3\mathrm{sgn}(x)}{c^3}\right.\right.$$
$$-\frac{3\left(b^4\mathrm{sgn}(x)-8\,ab^2c\mathrm{sgn}(x)+16\,a^2c^2\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}}$$
$$+\frac{\left(3\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-24\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^3\sqrt{c}-40\,a^{\frac{3}{2}}bc\right.}{128\,c^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sgn(x) + 3*b*sgn(x))*x + (b^2*c^2*
sgn(x) + 20*a*c^3*sgn(x))/c^3)*x - (3*b^3*c*sgn(x) - 20*a*b*c^2*sgn(x))/c^
3) - 3/128*(b^4*sgn(x) - 8*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) + 1/128*(3*b^4*lo
g(abs(b - 2*sqrt(a)*sqrt(c))) - 24*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c)))
 + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^3*sqrt(c) - 40
*a^(3/2)*b*c^(3/2))*sgn(x)/c^(5/2)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^3}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3, x)
```

3.43 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx$

3.43.1 Optimal result

Integrand size = 24, antiderivative size = 227

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx = \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^3-a^(3/2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/
(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/16*b*
(-12*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+
b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/8*(2*b*c*x+8*a*c+b^2)*(c*
x^4+b*x^3+a*x^2)^(1/2)/c/x
```

3.43.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(-3b(b^2-12ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(\sqrt{a+x(b\right.\right.}{48c^{3/2}\sqrt{x^2(a+}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(-3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*
b*c*x + 8*c*(4*a + c*x^2)) + 48*a^(3/2)*c*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*
(b + c*x)])/Sqrt[a]])))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.43.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1968, 1992, 27, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \; 1968$$

$$\frac{1}{2}\int \frac{(2a+bx)\sqrt{cx^4+bx^3+ax^2}}{x^2}dx+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 1992$$

$$\frac{1}{2}\left(\frac{\int \frac{16a^2c-b(b^2-12ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{4c}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bcx)}{4cx}\right)+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{16a^2c-b(b^2-12ac)x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bcx)}{4cx}\right)+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 1980$$

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\int \frac{16a^2c-b(b^2-12ac)x}{x\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bcx)}{4cx}\right)+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\downarrow$ 1269

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(16a^2c\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-b(b^2-12ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bcx}{4cx}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(16a^2c\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2b(b^2-12ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac}{4cx}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(16a^2c\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2}{4cx}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(-32a^2c\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{4}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

3.43. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(-16a^{3/2}c\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx}}{4}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

output
```
(a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^3) + (((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x
^2 + b*x^3 + c*x^4])/(4*c*x) + (x*Sqrt[a + b*x + c*x^2]*(-16*a^(3/2)*c*Arc
Tanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] - (b*(b^2 - 12*a*c)*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]))/(8*c*Sqrt[
a*x^2 + b*x^3 + c*x^4]))/2
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1968
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Simp[(n - q)*(p/(m + p*(2*n - q) + 1))   Int[x^(m + q)*
(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; Free
Q[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
 + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

3.43. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx$

3.43.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{16x^2\sqrt{c\,x^2+bx+a}\,c^{\frac{5}{2}}+48\ln(2)a^{\frac{3}{2}}c^{\frac{3}{2}}-48\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)a^{\frac{3}{2}}c^{\frac{3}{2}}+28c^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,bx+36\ln\left(2\sqrt{c\,x^2+bx+a}\right.}{48c^{\frac{3}{2}}}$
default	$-\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(48c^{\frac{5}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)-16(c\,x^2+bx+a)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,bx-48c^{\frac{5}{2}}\sqrt{c\,x^2+bx}\right.}{48x^3(c\,x^2+bx+a)^{\frac{3}{2}}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/48/c^(3/2)*(16*x^2*(c*x^2+b*x+a)^(1/2)*c^(5/2)+48*ln(2)*a^(3/2)*c^(3/2)-
48*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))*a^(3/2)*c^(3/2)+2
8*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b*x+36*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*
x+b)*a*b*c-3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^3+64*a*c^(3/2)*(c
*x^2+b*x+a)^(1/2)+6*c^(1/2)*(c*x^2+b*x+a)^(1/2)*b^2)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 791, normalized size of antiderivative = 3.48

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \left[\frac{48\,a^{\frac{3}{2}}c^2x\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-3\,(b^3-12\,ab\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*
sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)
*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*
c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*
x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2
*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c
*x)) + 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3
+ a*x^2))/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3
 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*
a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x +
 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-
a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*
c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(
c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) +
 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^
2))/(c^2*x)]
```

3.43.6 Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**4, x)
```

3.43.7 Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)
```

3.43.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} \, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^4} \, dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x)
```

3.44 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5} dx$

3.44.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5} dx = \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} - \frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}} + \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-(c*x^4+b*x^3+a*x^2)^(3/2)/x^4-3/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^
2+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*
(4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*
x+a)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/4*(2*c*x+3*b)*(c*x^4+b*x^3+
a*x^2)^(1/2)/x
```

3.44.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(-4a+x(5b+2cx))+24\sqrt{a}b\sqrt{c}x\operatorname{arcta}\right.}{8\sqrt{c}\sqrt{x^2(a+}$$

input

```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

output

```
(Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2
*c*x)) + 24*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]] - 3*(b^2 + 4*a*c)*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c
*x)]]))/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

3.44.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1967, 1992, 27, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{2}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^2}dx - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}$$

$$\Big\downarrow \text{1992}$$

$$\frac{3}{2}\left(\frac{\int \frac{c(4ab+(b^2+4ac)x)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c} + \frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right) - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\left(\frac{1}{4}\int \frac{4ab+(b^2+4ac)x}{\sqrt{cx^4+bx^3+ax^2}}dx + \frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right) - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}$$

$$\Big\downarrow \text{1980}$$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\int\frac{4ab+(b^2+4ac)x}{x\sqrt{cx^2+bx+a}}dx}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$\Big\downarrow$ 1269

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx+4ab\int\frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$\Big\downarrow$ 1092

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(2\left(4ac+b^2\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+4ab\int\frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+c}}{2x}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$\Big\downarrow$ 219

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(4ab\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)-$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$\Big\downarrow$ 1154

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-8ab\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3}}{2x}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$\Big\downarrow$ 219

3.44. $\displaystyle\int\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^5}dx$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(\frac{(4ac+b^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-4\sqrt{a}b\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3}}{2x}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

output
```
-((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4) + (3*(((3*b + 2*c*x)*Sqrt[a*x^2 + b*x
^3 + c*x^4])/(2*x) + (x*Sqrt[a + b*x + c*x^2]*(-4*Sqrt[a]*b*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]))/(4*Sqrt[a*x^2 + b*x^3 +
c*x^4])))/2
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

3.44. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx$

3.44.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{4c^{\frac{3}{2}}x^2\sqrt{c\,x^2+bx+a}-12\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)bx\sqrt{a}\,\sqrt{c}+12\ln(2)bx\sqrt{a}\,\sqrt{c}+10b\sqrt{c\,x^2+bx+a}\,x\sqrt{c}+12\ln\left(2\sqrt{c\,x^2+bx\right.}}{8x\sqrt{c}}$
risch	$-\frac{a\sqrt{x^2(c\,x^2+bx+a)}}{x^2}+\frac{\left(\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}+\frac{3a\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2}+\frac{c\sqrt{c\,x^2+bx+a}\,x}{2}+\frac{5\sqrt{c\,x^2+bx+a}\,b}{4}-\right.}{x\sqrt{c\,x^2+bx+a}}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(8c^{\frac{5}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}x^2+12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,a\,x^2-12c^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)bx-8(c\,x^2+bx+\right.}{8x^4(c}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*(4*c^(3/2)*x^2*(c*x^2+b*x+a)^(1/2)-12*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x/a^(1/2))*b*x*a^(1/2)*c^(1/2)+12*ln(2)*b*x*a^(1/2)*c^(1/2)+10*
b*(c*x^2+b*x+a)^(1/2)*x*c^(1/2)+12*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+
b)*a*c*x+3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^2*x-8*a*(c*x^2+b*x+
a)^(1/2)*c^(1/2))/x/c^(1/2)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.46

$$\int\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx=\left[\frac{12\sqrt{a}bcx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+3\,(b^2+4\,ac}{}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x -
4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*
sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x
^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*
a)*sqrt(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/16*(24*sqrt
(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(
a*c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3
+ 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4
*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))
/(c*x^2), 1/8*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-
c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^
3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x
 - 4*a*c))/(c*x^2)]
```

3.44.6 Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^5} \, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

3.44.7 Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^5} \, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)
```

3.44.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^5}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5, x)
```

3.45 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

3.45.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-1/2*(c*x^4+b*x^3+a*x^2)^(3/2)/x^5-3/8*(4*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)
/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/a^(1/2)/(c*x^4+b*x^3+a*x
^2)^(1/2)+3/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/
2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/4*(-2*c*x+b)*(c*x^4+b*x
^3+a*x^2)^(1/2)/x^2
```

3.45.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(3(b^2+4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\sqrt{a}\left((2a+x(5\right.\right.}{4\sqrt{a}x^3\sqrt{a+}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqr
t[a + x*(b + c*x)])/Sqrt[a]] - Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqrt[a + x
*(b + c*x)] + 6*b*Sqrt[c]*x^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])))/(4*Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])
```

3.45.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1967, 1988, 25, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{4}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^3}dx - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1988}$$

$$\frac{3}{4}\left(-\frac{1}{2}\int -\frac{b^2+4cxb+4ac}{\sqrt{cx^4+bx^3+ax^2}}dx - \frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right) - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{b^2+4cxb+4ac}{\sqrt{cx^4+bx^3+ax^2}}dx - \frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right) - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1980}$$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\int \frac{b^2+4cxb+4ac}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$\big\downarrow$ 1269

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+4bc\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$\big\downarrow$ 1092

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+8bc\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$\big\downarrow$ 219

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$\big\downarrow$ 1154

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-2\left(4ac+b^2\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$\big\downarrow$ 219

3.45. $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^6}\,dx$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{(4ac+b^2)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+}}{x^2}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

output
```
-1/2*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5 + (3*(-(((b - 2*c*x)*Sqrt[a*x^2 + b
*x^3 + c*x^4])/x^2) + (x*Sqrt[a + b*x + c*x^2]*(-(((b^2 + 4*a*c)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + 4*b*Sqrt[c]*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]))/(2*Sqrt[a*x^2 + b*x^3
 + c*x^4])))/4
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

3.45. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

3.45.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$-\frac{3\left(x^{2}\left(ac+\frac{b^{2}}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)-\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b\,x^{2}\sqrt{c}\,\sqrt{a}+\frac{\left(a^{\frac{3}{2}}+\left(-2c\,x^{2}+\frac{5}{2}bx\right)\sqrt{a}\right)\sqrt{c\,x^{2}+}}{3}\right.}{2\sqrt{a}\,x^{2}}$
risch	$-\frac{(5bx+2a)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{4x^{3}}+\frac{\left(c\sqrt{c\,x^{2}+bx+a}+\frac{3b\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2}-\frac{3\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)c}{2}-\right.}{x\sqrt{c\,x^{2}+bx+a}}$
default	$-\frac{(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\left(12a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)c^{\frac{5}{2}}x^{2}-2c^{\frac{5}{2}}(c\,x^{2}+bx+a)^{\frac{3}{2}}b\,x^{3}+3a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right.\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-3/2*(x^2*(a*c+1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/
2))-ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b*x^2*c^(1/2)*a^(1/2)+1/3*(a
^(3/2)+(-2*c*x^2+5/2*b*x)*a^(1/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*(a*c+1/4*b^2)
*x^2)/a^(1/2)/x^2
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.46

$$\int\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx=\left[\frac{12\,ab\sqrt{c}x^3\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+3\,(b^2+4\,ac)}{}\right.$$

$$-\frac{24\,ab\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-3\,(b^2+4\,ac)\sqrt{a}x^3\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+a}}{x^3}\right.}{16\,ax^3}$$

$$-\frac{12\,ab\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-3\,(b^2+4\,ac)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)-2\,\sqrt{}}{8\,ax^3}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x
^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sq
rt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b
*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4
*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/
2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*
c*x)) - 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b
*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*ar
ctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x
^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2)
)/(a*x^3), -1/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt
(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*
x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b
*x - 2*a^2))/(a*x^3)]
```

3.45.6 Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**6, x)
```

3.45.7 Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)
```

3.45.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6} \, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^6} \, dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x)
```

3.46 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

3.46.1 Optimal result

Integrand size = 24, antiderivative size = 257

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5} + \frac{b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{c^{3/2}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^6-1/4*b*(c*x^4+b*x^3+a*x^2)^(3/2)/a/x^5+1
/16*b*(-12*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(
c*x^2+b*x+a)^(1/2)/a^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+c^(3/2)*x*arctanh(1/2
*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a
*x^2)^(1/2)+1/8*(2*b*c*x-8*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

3.46.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.67

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx =$$
$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(3b(b^2-12ac)\,x^3\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+\sqrt{a}\left(\sqrt{a+x(b+cx)}(8a^2+3b^2x^2+2a\right.\right.}{24a^{3/2}x^4\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

output
```
-1/24*(Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(Sqrt[c
]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*
a^2 + 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) + 24*a*c^(3/2)*x^3*Log[b + 2*c*x -
 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])))/(a^(3/2)*x^4*Sqrt[a + x*(b + c*x)])
```

3.46.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1967, 1998, 27, 1988, 25, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{2}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^4}dx-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{2}\left(-\frac{\int \frac{(b^2-2cxb-8ac)\sqrt{cx^4+bx^3+ax^2}}{2x^3}dx}{2a}-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{2ax^5}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{(b^2-2cxb-8ac)\sqrt{cx^4+bx^3+ax^2}}{x^3}dx}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 1988$$

$$\frac{1}{2}\left(-\frac{-\frac{1}{2}\int -\frac{b(b^2-12ac)-16ac^2x}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}\,(-8ac+b^2+2bcx)}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{b(b^2-12ac)-16ac^2x}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 1980$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\int \frac{b\left(b^2-12ac\right)-16ac^2x}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 1269$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(b(b^2-12ac)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-16ac^2\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right.}{2ax^5}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 1092$$

3.46. $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^7}\,dx$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(b(b^2-12ac)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx-32ac^2\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b(ax^2+\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(b(b^2-12ac)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b(ax^2-\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(-2b(b^2-12ac)\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+c}}{x^2}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

3.46. $\int\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

output
```
-1/3*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6 + (-1/2*(b*(a*x^2 + b*x^3 + c*x^4)^
(3/2))/(a*x^5) - (-(((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/
x^2) + (x*Sqrt[a + b*x + c*x^2]*(-((b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x)/(
2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) - 16*a*c^(3/2)*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]))/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]))
/(4*a))/2
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.46. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.46. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

3.46.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{3\left(b x^3\left(ac-\frac{b^2}{12}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c x^2+bx+a}}{x\sqrt{a}}\right)-\frac{4c^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(2\sqrt{c x^2+bx+a}\,\sqrt{c}+2cx+b\right)x^3}{3}+\left(\frac{7x\left(\frac{16cx}{7}+b\right)a^{\frac{3}{2}}}{9}+\frac{\sqrt{a}\,b^2x^2}{6}+\frac{4a^{\frac{5}{2}}}{9}\cdots\right)\right)}{4a^{\frac{3}{2}}x^3}$
risch	$-\frac{\left(32ac\,x^2+3b^2x^2+14abx+8a^2\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{24x^4a}+\frac{\left(16a\,c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)-\frac{b\left(12ac-b^2\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\cdots}}{x}\right)}{\sqrt{a}}\cdots\right)}{16ax\sqrt{c\,x^2+bx+a}\cdots}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(32c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}a\,x^4-36c^{\frac{5}{2}}a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)b\,x^3+48c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,a^2x^4-2c^{\frac{5}{2}}\left(c\cdots\right)\right)}{\cdots}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-3/4*(b*x^3*(a*c-1/12*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^
(1/2))-4/3*c^(3/2)*a^(3/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*x^3+(
7/9*x*(16/7*c*x+b)*a^(3/2)+1/6*a^(1/2)*b^2*x^2+4/9*a^(5/2))*(c*x^2+b*x+a)^
(1/2)-ln(2)*(a*c-1/12*b^2)*x^3*b)/a^(3/2)/x^3
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.17

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^7}\,dx=\left[\frac{48\,a^2c^{\frac{3}{2}}x^4\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-3\,(b^3-12\,ab\cdots}{\cdots}\right.$$

$$-\frac{96\,a^2\sqrt{-c}cx^4\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+3\,(b^3-12\,abc)\sqrt{a}x^4\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx\cdots}}{x^3}\right)\cdots}{96\,a^2x^4}$$

$$-\frac{48\,a^2\sqrt{-c}cx^4\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+3\,(b^3-12\,abc)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+\cdots}{48\,a^2x^4}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x
^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)
*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4
+ b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2
*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c
)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*
b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 +
(3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2
*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^
2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt
(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(
a^2*x^4), -1/48*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)
*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/
(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x +
 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4)]
```

3.46.6 Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7} dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^7} dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**7, x)
```

3.46.7 Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7} dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^7} dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)
```

3.46.8 Giac [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
sage0*x
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^7}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x)
```

3.47 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx$

3.47.1 Optimal result

Integrand size = 24, antiderivative size = 197

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = &-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} \\ &+\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} \\ &-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}-\frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}\end{aligned}$$

output
```
-1/4*(c*x^4+b*x^3+a*x^2)^(3/2)/x^7-3/128*(-4*a*c+b^2)^2*arctanh(1/2*x*(b*x
+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)-1/32*(-12*a*c+b^2)*(c*x^4
+b*x^3+a*x^2)^(1/2)/a/x^3+1/64*b*(-20*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)
/a^2/x^2-1/8*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4
```

3.47.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(-\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}(8a^2-3b^2x^2+4ax(2b+5\right.}{64a^{5/2}x^5\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(-(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]*
(8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x))) + 3*(b^2 - 4*a*c)^2*x^4*ArcTanh
[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(64*a^(5/2)*x^5*Sqrt[a + x
*(b + c*x)])
```

3.47.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1967, 1988, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{8}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^5}dx-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1988}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \frac{b^2-4cxb-12ac}{x^2\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{3x^4}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1998}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\int \frac{b(3b^2-20ac)+2c(b^2-12ac)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{3x^4}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\int \frac{b(3b^2-20ac)+2c(b^2-12ac)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{3x^4}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1998}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{-\frac{\int \frac{3\left(b^2-4ac\right)^2}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+6cx)\sqrt{ax^2+bx^3}}{3x^4}\right.$$
$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}$$

$\downarrow$ 27

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{-\frac{3(b^2-4ac)^2\int\frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+6cx)\sqrt{}}{}\right.$$
$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}$$

$\downarrow$ 1951

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\frac{3(b^2-4ac)^2\int\frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\right.$$
$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}$$

$\downarrow$ 219

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\frac{3(b^2-4ac)^2\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+}{}\right.$$
$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

output
```
-1/4*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7 + (3*(-1/3*((b + 6*c*x)*Sqrt[a*x^2
+ b*x^3 + c*x^4])/x^4 + (-1/2*((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(a*x^3) - (-((b*(3*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) +
(3*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 +
 c*x^4])])/(2*a^(3/2)))/(4*a))/6))/8
```

3.47. $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^8}\,dx$

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{3\left(x^{4}\left(ac-\frac{b^{2}}{4}\right)^{2}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)+\left(\frac{b\,x^{2}(10cx+b)a^{\frac{3}{2}}}{12}+x\left(\frac{5cx}{3}+b\right)a^{\frac{5}{2}}-\frac{\sqrt{a}\,b^{3}x^{3}}{8}+\frac{2a^{\frac{7}{2}}}{3}\right)\sqrt{c\,x^{2}+bx+a}-\ln(2)x^{4}\right.}{8a^{\frac{5}{2}}x^{4}}$
risch	$-\frac{\left(20abc\,x^{3}-3b^{3}x^{3}+40a^{2}c\,x^{2}+2a\,b^{2}x^{2}+24a^{2}bx+16a^{3}\right)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{64x^{5}a^{2}}-\frac{3\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+}}{x}\right.}{128a^{\frac{5}{2}}x\sqrt{c\,x^{2}+bx+}}$
default	$-\frac{\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(48c^{2}a^{\frac{7}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)x^{4}+24c^{2}\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}ab\,x^{5}-24c\,a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+b}}{x}\right.\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-3/8*(x^4*(a*c-1/4*b^2)^2*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(
1/2))+(1/12*b*x^2*(10*c*x+b)*a^(3/2)+x*(5/3*c*x+b)*a^(5/2)-1/8*a^(1/2)*b^3
*x^3+2/3*a^(7/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^4*(a*c-1/4*b^2)^2)/a^(5/2)/x
^4
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.69

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\right.}{x^3}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (b^
2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(
a))/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^
2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 -
 8*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*
a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4
+ b*x^3 + a*x^2))/(a^3*x^5)]
```

3.47.6 Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**8, x)
```

3.47.7 Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)
```

3.47.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8}\, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^8}\, dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8, x)
```

3.48 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9} dx$

3.48.1 Optimal result

Integrand size = 24, antiderivative size = 249

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9} dx = -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} + \frac{3b(b^2-4ac)^2\,\mathrm{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}$$

output
```
-1/5*(c*x^4+b*x^3+a*x^2)^(3/2)/x^8+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*x*(b
*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(7/2)-1/80*(-8*a*c+b^2)*(c*x^
4+b*x^3+a*x^2)^(1/2)/a/x^4+1/320*b*(-28*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/
2)/a^2/x^3-1/640*(128*a^2*c^2-100*a*b^2*c+15*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2
)/a^3/x^2-3/40*(4*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5
```

3.48.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx =$$
$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(128a^4+15b^4x^4-10ab^2x^3(b+10cx)+16a^3x(11b+16cx)+8a\right.}{640a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

output
```
-1/640*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^
4 + 15*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*
a^2*x^2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(
Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(7/2)*x^6*Sqrt[a + x*(b +
 c*x)])
```

3.48.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1967, 1988, 27, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{10}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^6}dx-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1988}$$

$$\frac{3}{10}\left(\frac{1}{16}\int \frac{2(b^2-2cxb-8ac)}{x^3\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{10}\left(\frac{1}{8}\int \frac{b^2-2cxb-8ac}{x^3\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 1998$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{\int \frac{b(5b^2-28ac)+4c(b^2-8ac)x}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{\int \frac{b(5b^2-28ac)+4c(b^2-8ac)x}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 1998$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\int \frac{15b^4-100acb^2+2c\left(5b^2-28ac\right)xb+128a^2c^2}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\int \frac{15b^4-100acb^2+2c\left(5b^2-28ac\right)xb+128a^2c^2}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 1998$$

3.48. $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^9}\,dx$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{-\frac{\int \frac{15b(b^2-4ac)^2}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(b^2-8ac)\sqrt{ax^2+bx}}{3ax^4}\right.\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{-\frac{15b(b^2-4ac)^2\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(b^2-8ac)}{}\right.\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 1951$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\frac{15b(b^2-4ac)^2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\right.\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 219$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\frac{15b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(b^2}{}\right.\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

3.48. $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx$

output
```
-1/5*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8 + (3*(-1/4*((b + 4*c*x)*Sqrt[a*x^2
+ b*x^3 + c*x^4])/x^5 + (-1/3*((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(a*x^4) - (-1/2*(b*(5*b^2 - 28*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) -
 (-(((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*
x^2)) + (15*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^
2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))/(6*a))/8))/10
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.48.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\frac{3 x^5 b \left(ac-\frac{b^2}{4}\right)^2 \ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)}{16}+\frac{3\left(-\frac{x^2\left(16c^2x^2+7bcx+b^2\right)a^{\frac{5}{2}}}{15}+\frac{b^2x^3(10cx+b)a^{\frac{3}{2}}}{12}-\frac{22x\left(\frac{16cx}{11}+b\right)a^{\frac{7}{2}}}{15}-\frac{b^4x^4\sqrt{a}}{8}-16 \cdots\right)}{16}}{a^{\frac{7}{2}}x^5}$
risch	$-\frac{\left(128a^2c^2x^4-100a\,b^2c\,x^4+15b^4x^4+56a^2bc\,x^3-10a\,b^3x^3+256a^3c\,x^2+8a^2b^2x^2+176a^3bx+128a^4\right)\sqrt{x^2(c\,x^2+bx+a)}}{640x^6a^3}+\frac{3(\cdots}{\cdots}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(240c^2a^{\frac{7}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)b\,x^5+120c^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}a\,b^2x^6-120c\,a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2 \cdots}}{x}\right)\cdots\right)}{\cdots}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
3/16/a^(7/2)*(x^5*b*(a*c-1/4*b^2)^2*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1
/2))/x/a^(1/2))+(-1/15*x^2*(16*c^2*x^2+7*b*c*x+b^2)*a^(5/2)+1/12*b^2*x^3*(
10*c*x+b)*a^(3/2)-22/15*x*(16/11*c*x+b)*a^(7/2)-1/8*b^4*x^4*a^(1/2)-16/15*
a^(9/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^5*b*(a*c-1/4*b^2)^2)/x^5
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.58

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}x^6\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx\cdots}{x^3}\right.}{\cdots}\right.$$

$$-\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,(176\,a^4bx+128\,a^5+(15\,ab^4-100\cdots}{1280\,a^4x^6}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 +
 (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*s
qrt(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*
a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2
)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*
a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*
a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3
+ 8*(a^3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]
```

3.48.6 Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**9, x)
```

3.48.7 Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9} \, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^9} \, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)
```

3.48.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9} \, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^9} \, dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9, x)
```

3.49 $\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

3.49.1 Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{(3b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
1/8*(-4*a*c+3*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c
*x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(c*x^4+b*x^3+a*x^2
)^(1/2)/c-3/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x
```

3.49.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{x\left(2\sqrt{c}(-3b+2cx)(a+x(b+cx)) + (-3b^2+4ac)\sqrt{a+x(b+cx)}\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx}\right.\right.\right.}{8c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (-3*b^2 + 4*a*c)*Sqrt[a +
 x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*
c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.49.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1975, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{1975}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x(2a+3bx)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x(2a+3bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}$$

$$\Big\downarrow \text{1996}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{(3b^2-4ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{(3b^2-4ac)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}$$

input
```
Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - ((3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*
x) - ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c)
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1975
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - 2*n + q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(
p + 1)/(c*(m + p*q + 2*(n - q)*p + 1))), x] - Simp[1/(c*(m + p*q + 2*(n - q
)*p + 1))   Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q
 + (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && Rationa
lQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.49.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{4\sqrt{c\,x^{2}+bx+a}\,c^{\frac{3}{2}}x-6b\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}-4\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)ac+3\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b^{2}}{8c^{\frac{5}{2}}}$
risch	$-\frac{(-2cx+3b)(c\,x^{2}+bx+a)x}{4c^{2}\sqrt{x^{2}(c\,x^{2}+bx+a)}}-\frac{(4ac-3b^{2})\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)x\sqrt{c\,x^{2}+bx+a}}{8c^{\frac{5}{2}}\sqrt{x^{2}(c\,x^{2}+bx+a)}}$
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\left(4c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,x-6c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b-4\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)a\,c^{2}+3\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right.}{8\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{7}{2}}}$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(4*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x-6*b*(c*x^2+b*x+a)^(1/2)*c^(1/2)-4*ln(
2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*a*c+3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1
/2)+2*c*x+b)*b^2)/c^(5/2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.58

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= \left[-\frac{(3b^2-4ac)\sqrt{c}x\log\left(-\frac{8c^2x^3+8bcx^2-4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)-4\sqrt{cx^4+bx^3+ax^2}(2c^2x-3b}{16c^3x}\right.$$
$$\left.-\frac{(3b^2-4ac)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)-2\sqrt{cx^4+bx^3+ax^2}(2c^2x-3bc)}{8c^3x}\right]$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*sqrt
(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^
3 + b*c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(
c^3*x)]
```

3.49.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^3}{\sqrt{x^2(a+bx+cx^2)}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.49.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,x}{c\mathrm{sgn}\left(x\right)}-\frac{3\,b}{c^2\mathrm{sgn}\left(x\right)}\right)\\&\quad+\frac{\left(3\,b^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-4\,ac\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{8\,c^{\frac{5}{2}}}\\&\quad-\frac{\left(3\,b^2-4\,ac\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}\end{aligned}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*x/(c*sgn(x)) - 3*b/(c^2*sgn(x))) + 1/8*(3*b^2
*log(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) +
 6*sqrt(a)*b*sqrt(c))*sgn(x)/c^(5/2) - 1/8*(3*b^2 - 4*a*c)*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(5/2)*sgn(x))
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^3}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.50 $\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

3.50.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^
(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(c*x^4+b*x^3+a*x^2)^(1/2)/c/x
```

3.50.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx =\frac{x\left(2\sqrt{c}(a+x(b+cx))-b\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c
*x))])
```

3.50.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1964, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{1964}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{b\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3
 + c*x^4])
```

3.50.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1964
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Simp[b/(2*c)   Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c
*x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
 q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p
, q] && EqQ[m + p*(n - 1) - 1, 0]
```

3.50.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b-2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}{2c^{\frac{3}{2}}}$	50
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\left(2\sqrt{c\,x^{2}+bx+a}\,c^{\frac{3}{2}}-b\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)c\right)}{2\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{5}{2}}}$	88
risch	$\frac{(c\,x^{2}+bx+a)\,x}{c\sqrt{x^{2}(c\,x^{2}+bx+a)}}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)x\sqrt{c\,x^{2}+bx+a}}{2c^{\frac{3}{2}}\,\sqrt{x^{2}(c\,x^{2}+bx+a)}}$	93

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^(3/2)*(ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b-2*(c*x^2+b*x+a)^
(1/2)*c^(1/2))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.83

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,\sqrt{cx^4+bx^3+ax^2}c}{4\,c^2x}, \; b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+\cdots}}{2\,(\cdots}\right.\right.$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 +
a*x^2)*c)/(c^2*x)]
```

3.50.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.50.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^2}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\left(b\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}\sqrt{c}\right)\operatorname{sgn}(x)}{2c^{\frac{3}{2}}} + \frac{b\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2c^{\frac{3}{2}}\operatorname{sgn}(x)} + \frac{\sqrt{cx^2+bx+a}}{c\operatorname{sgn}(x)}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*sqrt(c))*sgn(x)/c^(3/2
) + 1/2*b*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(
3/2)*sgn(x)) + sqrt(c*x^2 + b*x + a)/(c*sgn(x))
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^2}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.51 $\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

3.51.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c
^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{x\sqrt{a+bx+cx^2}\log\left(b+2cx-2\sqrt{c}\sqrt{a+bx+cx^2}\right)}{\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
-((x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
])/(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))]))
```

3.51.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{ 1961}$$

$$\frac{x\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{2x\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

3.51.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

3.51.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)}{\sqrt{c}}$	29
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)}{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\sqrt{c}}$	65

input
```
int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.82

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)}{2\,\sqrt{c}},\ -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)}{c}\right]$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4
+ b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]
```

3.51.6 Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.51.7 Maxima [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)\operatorname{sgn}(x)}{\sqrt{c}} - \frac{\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{\sqrt{c}\operatorname{sgn}(x)}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(b - 2*sqrt(a)*sqrt(c)))*sgn(x)/sqrt(c) - log(abs(2*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*sqrt(c) + b))/(sqrt(c)*sgn(x))
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{x}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.52 $\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

3.52.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{2x\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqr
t[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

3.52.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$\Big\downarrow \text{ 1951}$$

$$-2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
-(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a]
)
```

3.52.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

3.52.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)}{\sqrt{a}}$	42
default	$-\frac{x\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\sqrt{a}}$	66

input
```
int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(ln(2)-ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(1/2)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$=\left[\frac{\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

3.52.6 Sympy [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} \, dx = \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} \, dx$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

3.52.7 Maxima [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} \, dx = \int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} \, dx$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} \, dx = -\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a} \operatorname{sgn}(x)}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.53 $\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$

3.53.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx=-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}+\frac{b\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}$$

output
```
1/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)-(
c*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

3.53.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx =\frac{-\sqrt{a}(a+x(b+cx))-bx\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
(-(Sqrt[a]*(a + x*(b + c*x))) - b*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]
*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(a^(3/2)*Sqrt[x^2*(a + x*(b + c*x))]
)
```

3.53.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1974, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}dx$$
$$\Big\downarrow \text{ 1974}$$
$$-\frac{b\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$
$$\Big\downarrow \text{ 1951}$$
$$\frac{b\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{b\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

input
```
Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
-(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqr
t[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

3.53.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1974
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*((a*x^q + b*x^n + c*x^(2*n - q))^(p +
 1)/(2*a*(n - q)*(p + 1))), x] - Simp[b/(2*a)   Int[x^(m + n - q)*(a*x^q +
b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ
[p, -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p +
 1)]
```

3.53.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{bx\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)-bx\ln\left(2\right)-2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{2a^{\frac{3}{2}}x}$	68
default	$-\frac{\sqrt{c\,x^{2}+bx+a}\,\left(2a^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}-b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)ax\right)}{2\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,a^{\frac{5}{2}}}$	88
risch	$-\frac{c\,x^{2}+bx+a}{a\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)x\sqrt{c\,x^{2}+bx+a}}{2a^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}$	97

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^(3/2)*(b*x*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))-b*x
*ln(2)-2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.52

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}a}{4\,a^2x^2},\ -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}a}{2\,a^2x^2}\right]$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

3.53.6 Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

3.53.7 Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^3+ax^2}\,x}\,dx$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)
```

3.53.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

3.54 $\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$

3.54.1 Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}$$

output
```
-1/8*(-4*a*c+3*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1
/2))/a^(5/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3+3/4*b*(c*x^4+b*x^3+a*x^2)
^(1/2)/a^2/x^2
```

3.54.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{-\sqrt{a}(2a-3bx)(a+x(b+cx))+(3b^2-4ac)\,x^2\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
(-(Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x))) + (3*b^2 - 4*a*c)*x^2*Sqrt[a +
 x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(
5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

3.54.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1976, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{1976}$$

$$\frac{\int -\frac{3b+2cx}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{3b+2cx}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{1998}$$

$$-\frac{-\frac{\int \frac{3b^2-4ac}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{(3b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{1951}$$

$$-\frac{\frac{(3b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(3b^2-4ac)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

input
```
Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^3) - ((-3*b*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(a*x^2) + ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*
x^2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a)
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1976
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
/(a*(m + p*q + 1))), x] - Simp[1/(a*(m + p*q + 1))   Int[x^(m + n - q)*(b*(
m + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
 q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && Eq
Q[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0
]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.54.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{(c\,x^2+bx+a)(-3bx+2a)}{4a^2x\sqrt{x^2(c\,x^2+bx+a)}}+\frac{(4ac-3b^2)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x\sqrt{c\,x^2+bx+a}}{8a^{\frac{5}{2}}\sqrt{x^2(c\,x^2+bx+a)}}$
pseudoelliptic	$\frac{4\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)ac\,x^2-3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)b^2x^2-4\ln(2)ac\,x^2+3\ln(2)b^2x^2-4a^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}+6b}{8a^{\frac{5}{2}}x^2}$
default	$-\frac{\sqrt{c\,x^2+bx+a}\left(-6a^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,bx-4c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)a^2x^2+3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)a\,b^2x^2+4a^{\frac{5}{2}}\right.}{8x\sqrt{c\,x^4+b\,x^3+a\,x^2}\,a^{\frac{7}{2}}}$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)*(-3*b*x+2*a)/a^2/x/(x^2*(c*x^2+b*x+a))^(1/2)+1/8*(4*a*c
-3*b^2)/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x/(x^2*(c*x^
2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$=\left[-\frac{(3\,b^2-4\,ac)\sqrt{a}x^3\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(3\,abx-\right.}{16\,a^3x^3}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)
*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/
(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*
a^2))/(a^3*x^3)]
```

3.54.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{1}{x^2\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

3.54.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)
```

3.54.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

3.55 $\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}} \, dx$

3.55.1 Optimal result

Integrand size = 24, antiderivative size = 262

$$\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}} \, dx = \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3(b^2-4ac)x} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{3(5b^2-4ac)x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(-4*a*c+5*b^2)*
x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c
^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(
1/2)/c^2/(-4*a*c+b^2)-1/4*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3
/(-4*a*c+b^2)/x-2*b*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{x\left(2\sqrt{c}(4a^2c(-13b+6cx)+b^2x(15b^2+5bcx-2c^2x^2)+a(15b^3-62b^2cx-20\right.}{8c^{7/2}(}$$

input
```
Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x
^2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) + 3*(5*b^4 - 24*
a*b^2*c + 16*a^2*c^2)*Sqrt[a + x*(b + c*x)]*Log[c^3*(b + 2*c*x - 2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b +
c*x))])
```

3.55.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1970, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \; 1970$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{3x^3(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \; 27$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\int \frac{x^3(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \; 1996$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int \frac{x^2\left(4ab+\left(5b^2-12ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{3c}\right)}{b^2-4ac}$$

$$\Big\downarrow \; 27$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int \frac{x^2\left(4ab+\left(5b^2-12ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{6c}\right)}{b^2-4ac}$$

$$\Big\downarrow \; 1996$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{6c}\right)}{b^2-4ac}$$

$$\Big\downarrow \; 27$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}}{6c}\right)}{b^2-4ac}$$

$$\Big\downarrow \; 1996$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}}{6c}\right)}{b^2-4ac}$$

$$\Big\downarrow \; 27$$

3.55. $\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int\frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}}{6c}\right)}{b^2-4ac}$$

$\big\downarrow$ 1961

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}\right)}{b^2-4ac}$$

$\big\downarrow$ 1092

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}\right.}{b^2-4ac}$$

$\big\downarrow$ 219

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}\right.}{b^2-4ac}$$

input
```
Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

3.55. $\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

output
```
(2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (6*((b*x
*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c) - (((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^
3 + c*x^4])/(2*c) - ((b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*
x) - (3*(b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3
+ c*x^4]))/(4*c))/(6*c)))/(b^2 - 4*a*c)
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1970
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1
/((n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
 - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ
[p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{48\left(\left(-\frac{5}{24}b^3x^2+\frac{31}{12}b^2ax+\frac{13}{6}a^2b\right)c^{\frac{3}{2}}+\left(\frac{1}{12}b^2x^3+\frac{5}{6}ab\,x^2-a^2x\right)c^{\frac{5}{2}}-\frac{a\,c^{\frac{7}{2}}x^3}{3}-\frac{5b^3\sqrt{c}\,(bx+a)}{8}+\frac{\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)\sqrt{c\,x}}{16}\right.}{\sqrt{c\,x^2+bx+a}\,c^{\frac{7}{2}}\left(32ac-8b^2\right)}$
default	$\frac{x^3\left(c\,x^2+bx+a\right)\left(16c^{\frac{9}{2}}a\,x^3-4c^{\frac{7}{2}}b^2x^3-40c^{\frac{7}{2}}ab\,x^2+48c^{\frac{7}{2}}a^2x+10c^{\frac{5}{2}}b^3x^2-124c^{\frac{5}{2}}a\,b^2x-104c^{\frac{5}{2}}a^2b+30c^{\frac{3}{2}}b^4x+30c^{\frac{3}{2}}a\,b^3-48\right.}{8c}$
risch	$-\frac{\left(-2cx+7b\right)\left(c\,x^2+bx+a\right)x}{4c^3\sqrt{x^2\left(c\,x^2+bx+a\right)}}-\frac{\left(\frac{8c\,a^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{14b^2a(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\left(-4abc-7b^3\right)\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(}{c\left(4ac-b^2\right.}\right.\right.}{}$

input
```
int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-48/(c*x^2+b*x+a)^(1/2)/c^(7/2)*((-5/24*b^3*x^2+31/12*b^2*a*x+13/6*a^2*b)*
c^(3/2)+(1/12*b^2*x^3+5/6*a*b*x^2-a^2*x)*c^(5/2)-1/3*a*c^(7/2)*x^3-5/8*b^3
*c^(1/2)*(b*x+a)+1/16*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*(c*x^2+b*x
+a)^(1/2)*(16*a^2*c^2-24*a*b^2*c+5*b^4))/(32*a*c-8*b^2)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.35

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^3+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^2+(5\,ab^4-\right.}{}\right.$$

$$-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^3+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^2+(5\,ab^4-24\,a^2b^2c+16\,a^3c^2)x\right)\sqrt{-c}\,\mathrm{arcta}}{8\left((b^2c^5-\right.}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(c)*lo
g(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt
(c) + (b^2 + 4*a*c)*x)/x) + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*
a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a
^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^
4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*
b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a
*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3
*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^
2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))
/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2
*c^5)*x)]
```

3.55.6 Sympy [F]

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^7}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.55.7 Maxima [F]

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\int \frac{x^7}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.21

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\frac{\left(15\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\,\sqrt{\right.\right.}{8\left(b^2c^{\frac{7}{2}}-4\,ac^{\frac{9}{2}}\right)}$$
$$+\frac{\left(\left(\frac{2\left(b^2c^2-4\,ac^3\right)x}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}-\frac{5\left(b^3c-4\,abc^2\right)}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,b^4-62\,ab^2c+24\,a^2c^2}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,ab^3-52\,a^2bc}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}}{4\sqrt{cx^2+bx+a}}$$
$$-\frac{3\left(5\,b^2-4\,ac\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(15*b^4*log(abs(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(b - 2*sqr
t(a)*sqrt(c))) + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b
^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))*sgn(x)/(b^2*c^(7/2) - 4*a*c^(9/2)) + 1
/4*(((2*(b^2*c^2 - 4*a*c^3)*x/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)) - 5*(b^3*c
 - 4*a*b*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*b^4 - 62*a*b^2*c
+ 24*a^2*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*a*b^3 - 52*a^2*b*
c)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))/sqrt(c*x^2 + b*x + a) - 3/8*(5*b^2 -
 4*a*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(7/
2)*sgn(x))
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.56 $\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.56.1 Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2(b^2-4ac)\,x} - \frac{3bx\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
2*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/2*b*x*arctanh(1/2
*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4
+b*x^3+a*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)+(-8*a*c+3
*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/(-4*a*c+b^2)/x
```

3.56.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{x\left(2\sqrt{c}(8a^2c-b^2x(3b+cx)+a(-3b^2+10bcx+4c^2x^2))-3b(b^2-4ac)\sqrt{a+\cdots}\right.}{2c^{5/2}(-b^2+4ac)\sqrt{x^2(a+x(b\cdots}}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*
x^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqr
t[c]*Sqrt[a + x*(b + c*x)]])))/(2*c^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(
b + c*x))])
```

3.56.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1970, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1970}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{2x^2(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\int \frac{x^2(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1996}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab+\left(3b^2-8ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab+\left(3b^2-8ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{1996}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int\frac{3b(b^2-4ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3b(b^2-4ac)\int\frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1961

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3bx(b^2-4ac)\sqrt{a+bx+cx^2}\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1092

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3bx(b^2-4ac)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 219

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3bx(b^2-4ac)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}\right)}{b^2-4ac}$$

input
```
Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

3.56. $\int\frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

output
```
(2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (4*((b*S
qrt[a*x^2 + b*x^3 + c*x^4])/(2*c) - (((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])/(c*x) - (3*b*(b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 +
c*x^4]))/(4*c)))/(b^2 - 4*a*c)
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1970
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1
/((n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
 - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ
[p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$-\frac{6\left(\frac{\left(\frac{1}{2}b^{2}x^{2}-5abx-4a^{2}\right)c^{\frac{3}{2}}}{3}-\frac{2c^{\frac{5}{2}}a\,x^{2}}{3}+\frac{b\left(2\left(b^{2}x+ab\right)\sqrt{c}+\sqrt{c\,x^{2}+bx+a}\,\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)\left(4ac-b^{2}\right)\right)}{4}\right)}{c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,\left(4ac-b^{2}\right)}$
default	$\frac{x^{3}\left(c\,x^{2}+bx+a\right)\left(8a\,c^{\frac{7}{2}}x^{2}-2c^{\frac{5}{2}}b^{2}x^{2}+20c^{\frac{5}{2}}abx-6c^{\frac{3}{2}}b^{3}x+16c^{\frac{5}{2}}a^{2}-6c^{\frac{3}{2}}a\,b^{2}-12\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{c\,x^{2}+bx+a}\,a\cdots\right.}{2c^{\frac{7}{2}}\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(4ac-b^{2}\right)}$
risch	$\frac{\left(c\,x^{2}+bx+a\right)x}{c^{2}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}+\frac{\left(\frac{3bx}{2c^{2}\sqrt{c\,x^{2}+bx+a}}-\frac{b^{2}}{4c^{3}\sqrt{c\,x^{2}+bx+a}}-\frac{b^{3}x}{2c^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{b^{4}}{4c^{3}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{3b\ln\left(\frac{b}{2}+\cdots\right)}{\cdots}\cdots\right.}{\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}$

input
```
int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6/c^(5/2)/(c*x^2+b*x+a)^(1/2)*(1/3*(1/2*b^2*x^2-5*a*b*x-4*a^2)*c^(3/2)-2/
3*c^(5/2)*a*x^2+1/4*b*(2*(b^2*x+a*b)*c^(1/2)+(c*x^2+b*x+a)^(1/2)*ln(2*(c*x
^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*(4*a*c-b^2)))/(4*a*c-b^2)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.42

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^3+(b^4-4\,ab^2c)x^2+(ab^3-4\,a^2bc)x\right)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx^2-}{}\right.}{4\left((b^2c^4-4\right.}\right.$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*
b*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2
)*x))/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 -
4*a^2*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (
a*b^3 - 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c
^2)*x))/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3
- 4*a^2*c^4)*x)]
```

3.56.6 Sympy [F]

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^6}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.56.7 Maxima [F]

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^6}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.15

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(b^2c^{\frac{5}{2}}-4\,ac^{\frac{7}{2}}\right)} + \frac{\left(\frac{(b^2c-4\,ac^2)x}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}\right)x+\frac{3\,ab^2-8\,a^2c}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}}{\sqrt{cx^2+bx+a}} + \frac{3\,b\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}(x)}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(
a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(b^2*c^(
5/2) - 4*a*c^(7/2)) + (((b^2*c - 4*a*c^2)*x/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(
x)) + (3*b^3 - 10*a*b*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))*x + (3*a*b^2 -
 8*a^2*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))/sqrt(c*x^2 + b*x + a) + 3/2*b
*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(5/2)*sgn(
x))
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax^2 + bx^3 + cx^4)^{3/2}}\, dx = \int \frac{x^6}{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}\, dx$$

input
```
int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.57 $\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

3.57.1 Optimal result

Integrand size = 24, antiderivative size = 153

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x} + \frac{x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
2*x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3
+a*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)/x
```

3.57.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = -\frac{x\left(2\sqrt{c}\left(-ab-b^2x+2acx\right)+\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{3/2}\left(-b^2+4ac\right)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
-((x*(2*Sqrt[c]*(-(a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b +
 c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(
-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]))
```

3.57.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1970, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1970}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 1996}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{(b^2-4ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{(b^2-4ac)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{ 1961}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}\right)}{b^2-4ac}$$

$$\Big\downarrow 219$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}\right)}{b^2-4ac}$$

input
```
Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*((b*S
qrt[a*x^2 + b*x^3 + c*x^4])/(c*x) - ((b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a
*x^2 + b*x^3 + c*x^4])))/(b^2 - 4*a*c)
```

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1970
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1
/((n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
 - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ
[p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.57.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}+\frac{b(bx+2a)}{\sqrt{c\,x^{2}+bx+a}\,c(4ac-b^{2})}+\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)}{c^{\frac{3}{2}}}$
default	$-\frac{x^{3}\left(c\,x^{2}+bx+a\right)\left(4c^{\frac{5}{2}}ax-2c^{\frac{3}{2}}b^{2}x-2c^{\frac{3}{2}}ab-4\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{c\,x^{2}+bx+a}\,a\,c^{2}+\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right.\right.}{c^{\frac{5}{2}}\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(4ac-b^{2}\right)}$

input
```
int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x/c/(c*x^2+b*x+a)^(1/2)+b*(b*x+2*a)/(c*x^2+b*x+a)^(1/2)/c/(4*a*c-b^2)+1/c
^(3/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.71

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{((b^2c-4ac^2)x^3+(b^3-4abc)x^2+(ab^2-4a^2c)x)\sqrt{c}\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+\ldots}}{\ldots}\right)\ldots}{2((b^2c^3-4ac^4)x^3+(b^3c^2-4\ldots}\right.$$

$$-\frac{((b^2c-4ac^2)x^3+(b^3-4abc)x^2+(ab^2-4a^2c)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)+2\sqrt{cx^4+bx^3+\ldots}}{(b^2c^3-4ac^4)x^3+(b^3c^2-4abc^3)x^2+(ab^2c^2-4a^2c^3)x}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*
sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c
 + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*
x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*
c)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3
*c^2 - 4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]
```

3.57.6 Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^5}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.57.7 Maxima [F]

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{x^5}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\frac{\left(b^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-4\,ac\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{b^2c^{\frac{3}{2}}-4\,ac^{\frac{5}{2}}}-\frac{2\left(\frac{ab}{b^2c\,\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}+\frac{(b^2-2\,ac)x}{b^2c\,\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}}-\frac{\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
(b^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c)
)) + 2*sqrt(a)*b*sqrt(c))*sgn(x)/(b^2*c^(3/2) - 4*a*c^(5/2)) - 2*(a*b/(b^2
*c*sgn(x) - 4*a*c^2*sgn(x)) + (b^2 - 2*a*c)*x/(b^2*c*sgn(x) - 4*a*c^2*sgn(
x)))/sqrt(c*x^2 + b*x + a) - log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) + b))/(c^(3/2)*sgn(x))
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{x^5}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.58 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.58.1 Optimal result

Integrand size = 24, antiderivative size = 40

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

output
```
2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.58.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1963}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1963}$$

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

3.58.3.1 Defintions of rubi rules used

rule 1963
```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[x^((n - 1)/2)*((4*a + 2*b*x)/((b^2 - 4*a*c)*Sqrt[a*x^
(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3
*n - 1)/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

3.58.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{-2bx-4a}{\sqrt{c\,x^2+bx+a}\,(4ac-b^2)}$	34
gosper	$-\frac{2(c\,x^2+bx+a)(bx+2a)x^3}{(4ac-b^2)(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}$	53
default	$-\frac{2(c\,x^2+bx+a)(bx+2a)x^3}{(4ac-b^2)(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}$	53
trager	$-\frac{2(bx+2a)\sqrt{c\,x^4+b\,x^3+a\,x^2}}{(c\,x^2+bx+a)x(4ac-b^2)}$	55

input
```
int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*b*x-4*a)/(c*x^2+b*x+a)^(1/2)/(4*a*c-b^2)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

3.58.6 Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^4}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.58.7 Maxima [F]

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^4}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\left(\frac{bx}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}+\frac{2\,a}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}} - \frac{4\,\sqrt{a}\mathrm{sgn}(x)}{b^2-4\,ac}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*(b*x/(b^2*sgn(x) - 4*a*c*sgn(x)) + 2*a/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt
(c*x^2 + b*x + a) - 4*sqrt(a)*sgn(x)/(b^2 - 4*a*c)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{\left(\frac{4\,a\,c}{4\,a\,c^2-b^2\,c}+\frac{2\,b\,c\,x}{4\,a\,c^2-b^2\,c}\right)\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x\,\left(c\,x^2+b\,x+a\right)}$$

input
```
int(x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
-(((4*a*c)/(4*a*c^2 - b^2*c) + (2*b*c*x)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3
 + c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

3.59 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.59.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-2*x*(2*c*x+b)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.59.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1962}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 1962}$$
$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

3.59.3.1 Defintions of rubi rules used

rule 1962
```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[-2*x^((n - 1)/2)*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, 3
*((n - 1)/2)] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{4cx+2b}{\sqrt{c\,x^{2}+bx+a}\,(4ac-b^{2})}$	33
gosper	$\frac{2(c\,x^{2}+bx+a)(2cx+b)x^{3}}{(4ac-b^{2})(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	52
default	$\frac{2(c\,x^{2}+bx+a)(2cx+b)x^{3}}{(4ac-b^{2})(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	52
trager	$\frac{2(2cx+b)\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}{(c\,x^{2}+bx+a)x(4ac-b^{2})}$	54

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/(c*x^2+b*x+a)^(1/2)*(2*c*x+b)/(4*a*c-b^2)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

3.59.6 Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(x^2\,(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.59.7 Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.90

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\sqrt{a}b\mathrm{sgn}(x)}{ab^2-4a^2c} - \frac{2\left(\frac{2cx}{b^2\mathrm{sgn}(x)-4ac\mathrm{sgn}(x)}+\frac{b}{b^2\mathrm{sgn}(x)-4ac\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(a)*b*sgn(x)/(a*b^2 - 4*a^2*c) - 2*(2*c*x/(b^2*sgn(x) - 4*a*c*sgn(x)
) + b/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt(c*x^2 + b*x + a)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\left(\frac{4c^2x}{4ac^2-b^2c}+\frac{2bc}{4ac^2-b^2c}\right)\sqrt{cx^4+bx^3+ax^2}}{x\,(cx^2+bx+a)}$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
(((4*c^2*x)/(4*a*c^2 - b^2*c) + (2*b*c)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3
+ c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

3.60 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.60.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

output
```
-arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x*(b
*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x\left(\sqrt{a}(b^2-2ac+bcx)+(b^2-4ac)\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{a^{3/2}(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(-2*x*(Sqrt[a]*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]
*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(3/2)*(-b^2 + 4
*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

3.60.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1969, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 1969$$

$$\frac{\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{a}+\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \ 1951$$

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}$$

$$\Big\downarrow \ 219$$

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

input
```
Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
- ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)
```

3.60.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1969
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r,
2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n,
0] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, (-(n - q))*(2*p
+ 3)]
```

3.60.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{4\left(-a^{\frac{3}{2}} c+\frac{b\sqrt{a}\,(c x+b)}{2}+\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)\right)\sqrt{c\,x^{2}+bx+a}\,\left(ac-\frac{b^{2}}{4}\right)\right)}{\sqrt{c\,x^{2}+bx+a}\,a^{\frac{3}{2}}\,(4ac-b^{2})}$
default	$\frac{x^{3}\left(c\,x^{2}+bx+a\right)\left(-2a^{\frac{3}{2}} bcx+4a^{\frac{5}{2}} c-2a^{\frac{3}{2}} b^{2}-4\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{c\,x^{2}+bx+a}\,a^{2} c+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\right.}{\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} a^{\frac{5}{2}}\,(4ac-b^{2})}$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/(c*x^2+b*x+a)^(1/2)*(-a^(3/2)*c+1/2*b*a^(1/2)*(c*x+b)+(-ln(2)+ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(c*x^2+b*x+a)^(1/2)*(a*c-1/4
*b^2))/a^(3/2)/(4*a*c-b^2)
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(84) = 168$.

Time = 0.31 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.37

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x)\sqrt{a}\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3}{}\right.}{2\,((a^2b^2c-4\,a^3c^2)x^3+(a^2b^3-}\right.$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*
sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x
^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b
*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b
*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c
)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^
2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]
```

3.60.6 Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^2}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.60.7 Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(84) = 168$.

Time = 0.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2\left(ab^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4\,a^2c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}b^2-2\,\sqrt{-a}a^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2b^2-4\,\sqrt{-a}a^3c} + \frac{2\left(\frac{abcx\mathrm{sgn}(x)}{a^2b^2-4\,a^3c}+\frac{ab^2\mathrm{sgn}(x)-2\,a^2c\mathrm{sgn}(x)}{a^2b^2-4\,a^3c}\right)}{\sqrt{cx^2+bx+a}} + \frac{2\,\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(sqrt(a)/sqrt(-a)) + sq
rt(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)*sgn(x)/(sqrt(-a)*a^2*b^2 - 4*sq
rt(-a)*a^3*c) + 2*(a*b*c*x*sgn(x)/(a^2*b^2 - 4*a^3*c) + (a*b^2*sgn(x) - 2*
a^2*c*sgn(x))/(a^2*b^2 - 4*a^3*c))/sqrt(c*x^2 + b*x + a) + 2*arctan(-(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a*sgn(x))
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^{3/2}}\, dx = \int \frac{x^2}{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}\, dx$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.61 $\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\, dx$

3.61.1 Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\, dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2} + \frac{3b\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}$$

output
```
3/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)+2
*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(-8*a*c+3*b^2)
*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^2
```

3.61.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.93

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\, dx = \frac{\sqrt{a}(-4a^2c+3b^2x(b+cx)+a(b^2-10bcx-8c^2x^2))+3b(b^2-4ac)\,x\sqrt{a+x(}}{a^{5/2}(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2)) +
 3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x
*(b + c*x)])/Sqrt[a]])/(a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]
)
```

3.61.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1971, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1971}$$

$$\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{3b^2+2cxb-8ac}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3b^2+2cxb-8ac}{x\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{\int \frac{3b(b^2-4ac)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1951}$$

$$\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3b(b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) +
(-(((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (3*b*(b^2 - 4*
a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*
a^(3/2)))/(a*(b^2 - 4*a*c))
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.61.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{\frac{\left(-4c^{2}x^{2}-5bcx+\frac{1}{2}b^{2}\right)a^{\frac{3}{2}}}{2}-a^{\frac{5}{2}}c+\frac{3x\left(\frac{b\sqrt{a}\,(cx+b)}{2}+\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)\right)\sqrt{c\,x^{2}+bx+a}\,\left(ac-\frac{b^{2}}{4}\right)\right)b}{2}}{\sqrt{c\,x^{2}+bx+a}\,a^{\frac{5}{2}}x\left(ac-\frac{b^{2}}{4}\right)}$
default	$-\frac{x^{2}(c\,x^{2}+bx+a)\,(16a^{\frac{5}{2}}c^{2}x^{2}-6a^{\frac{3}{2}}b^{2}c\,x^{2}+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^{3}x-12\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{c\,x^{2}+bx+a}\,a^{2}bcx+3\ln(}{2(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{7}{2}}(4ac-b^{2})}$
risch	$-\frac{c\,x^{2}+bx+a}{a^{2}\sqrt{x^{2}(c\,x^{2}+bx+a)}}+\frac{(\frac{2b^{2}cx}{a^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}+\frac{b^{3}}{a^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{4c^{2}x}{a\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{2cb}{a\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+b}}}{\sqrt{x^{2}(c\,x^{2}+bx+a)}}$

input
```
int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2/(c*x^2+b*x+a)^(1/2)/a^(5/2)*(1/3*(-4*c^2*x^2-5*b*c*x+1/2*b^2)*a^(3/2)-
2/3*a^(5/2)*c+x*(1/2*b*a^(1/2)*(c*x+b)+(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^
2+b*x+a)^(1/2))/x/a^(1/2)))*(c*x^2+b*x+a)^(1/2)*(a*c-1/4*b^2))*b)/x/(a*c-1
/4*b^2)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.44

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^4+(b^4-4\,ab^2c)x^3+(ab^3-4\,a^2bc)x^2\right)\sqrt{a}\log\left(-\frac{8\,abx^2+(b^2+\cdots}{\cdots}\right)\cdots}{4\left((a^3b^2c-\cdots\right)}\right.$$

$$-\frac{3\left((b^3c-4\,abc^2)x^4+(b^4-4\,ab^2c)x^3+(ab^3-4\,a^2bc)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\sqrt{cx^4+\cdots}}{2\left((a^3b^2c-4\,a^4c^2)x^4+(a^3b^3-4\,a^4bc)x^3+(a^4b^2\cdots\right)}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*
b*c)*x^2)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2
*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*
b^2 - 4*a^5*c)*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*
x^3 + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 -
 10*a^2*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3
+ (a^4*b^2 - 4*a^5*c)*x^2)]
```

3.61.6 Sympy [F]

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \int \frac{x}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

3.61.7 Maxima [F]

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.61.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x}{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}\,dx$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.62 $\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.62.1 Optimal result

Integrand size = 20, antiderivative size = 209

$$\begin{aligned}\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx &= \frac{2(b^2-2ac+bcx)}{a\,(b^2-4ac)\,x\sqrt{ax^2+bx^3+cx^4}} \\ &- \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2\,(b^2-4ac)\,x^3} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3\,(b^2-4ac)\,x^2} \\ &- \frac{3(5b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}\end{aligned}$$

output
```
-3/8*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1
/2))/a^(7/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^3+a*x^2)^(1/2
)-1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^3+1/4*b
*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^2
```

3.62.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{a}(-8a^3c-15b^3x^2(b+cx)+2a^2(b^2+10bcx-12c^2x^2)+abx(-5b^2+62bcx+52c^2x^2))-3(5b^4-24ab^2}{4a^{7/2}\,(b^2-4ac)\,x\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(Sqrt[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x - 1
2*c^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2
*c + 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x
*(b + c*x)])/Sqrt[a]])/(a^(7/2)*(b^2 - 4*a*c)*x*Sqrt[x^2*(a + x*(b + c*x))
])
```

3.62.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1954, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1954}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{5b^2+4cxb-12ac}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{5b^2+4cxb-12ac}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{ 1998}$$

$$\frac{-\frac{\int \frac{b\left(15b^2-52ac\right)+2c\left(5b^2-12ac\right)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{b\left(15b^2-52ac\right)+2c\left(5b^2-12ac\right)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{ 1998}$$

$$-\frac{-\frac{-\frac{\int \frac{3(b^2-4ac)(5b^2-4ac)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{3(b^2-4ac)(5b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 1951$$

$$-\frac{-\frac{\frac{3(b^2-4ac)(5b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\frac{3(b^2-4ac)(5b^2-4ac)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])
+ (-1/2*((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) - (-((b*(15
*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (3*(b^2 - 4*a*c)*(5
*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^
4])])/(2*a^(3/2)))/(4*a))/(a*(b^2 - 4*a*c))
```

3.62. $\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1954
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[(-x^(-q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q + b*x^n + c*x
^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*(n -
 q)*(p + 1)*(b^2 - 4*a*c))   Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1
)*(b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && Eq
Q[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ
[p, -1]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{5xb\left(-\frac{52}{5}c^2x^2-\frac{62}{5}bcx+b^2\right)a^{\frac{3}{2}}}{4}+6\left(-c^2x^2+\frac{5}{6}bcx+\frac{1}{12}b^2\right)a^{\frac{5}{2}}-2a^{\frac{7}{2}}c+6\left(-\frac{5b^3(cx+b)\sqrt{a}}{8}+\sqrt{c\,x^2+bx+a}\left(-\ln(2)+\ln\left(\frac{2a+bx+2\cdots}{}\right.\right.\right.}{a^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,(4ac-b^2)x^2}$
default	$-\frac{x(c\,x^2+bx+a)\left(-104a^{\frac{5}{2}}b\,c^2x^3+30a^{\frac{3}{2}}b^3c\,x^3+48a^{\frac{7}{2}}c^2x^2-124a^{\frac{5}{2}}b^2c\,x^2+30a^{\frac{3}{2}}b^4x^2-48\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\sqrt{c\,x\cdots}\right.}{}$
risch	$-\frac{(c\,x^2+bx+a)(-7bx+2a)}{4a^3x\sqrt{x^2(c\,x^2+bx+a)}}+\frac{\left(-\frac{2b^3cx}{a^3\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{b^4}{a^3\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{6c^2bx}{a^2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{3\cdots}{a^2\left(4ac-b^2\cdots\right.}\right.}{}$

input
```
int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
6/(c*x^2+b*x+a)^(1/2)*(-5/24*x*b*(-52/5*c^2*x^2-62/5*b*c*x+b^2)*a^(3/2)+(-
c^2*x^2+5/6*b*c*x+1/12*b^2)*a^(5/2)-1/3*a^(7/2)*c+(-5/8*b^3*(c*x+b)*a^(1/2
)+(c*x^2+b*x+a)^(1/2)*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
/a^(1/2)))*(a*c-5/4*b^2)*(a*c-1/4*b^2))*x^2)/a^(7/2)/(4*a*c-b^2)/x^2
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.01

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\left[-\frac{3\left((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^5+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^4+(5\,ab^4-\cdots\right.}{}\right.$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(a)*
log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*
a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 -
 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 +
 (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*c -
24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 +
 (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2
*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*
b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 +
a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^
2 - 4*a^6*c)*x^3)]
```

3.62.6 Sympy [F]

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{(ax^2+bx^3+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

3.62.7 Maxima [F]

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)
```

3.62.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.63 $\int \frac{1}{x(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.63.1 Optimal result

Integrand size = 24, antiderivative size = 271

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4\left(b^2-4ac\right)x^2} + \frac{5b(7b^2-12ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}$$

output
```
5/16*b*(-12*a*c+7*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)
^(1/2))/a^(9/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^3+a*x^2)
^(1/2)-1/3*(-16*a*c+7*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^4+
1/12*b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^3-1/
24*(256*a^2*c^2-460*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^4/(-4*a*c
+b^2)/x^2
```

3.63.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{a}(-32a^4c+105b^4x^3(b+cx)+5ab^2x^2(7b^2-106bcx-92c^2x^2)+8a^3(b^2+$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*c
*x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 -
86*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) + 15*b*(7*b^4 - 40*a*b^2*c + 48
*a^2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c
*x)])/Sqrt[a]])/(24*a^(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))]
)
```

3.63.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1971, 27, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1971}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{7b^2+6cxb-16ac}{2x^3\sqrt{cx^4+bx^3+ax^2}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{7b^2+6cxb-16ac}{x^3\sqrt{cx^4+bx^3+ax^2}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{ 1998}$$

$$\frac{-\frac{\int \frac{b\left(35b^2-116ac\right)+4c\left(7b^2-16ac\right)x}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{b\left(35b^2-116ac\right)+4c\left(7b^2-16ac\right)x}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)xb+256a^2c^2}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)xb+256a^2c^2}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2\ldots}}{3ax^4}}{a\left(b^2-4ac\right)}$$
$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1951}$$

3.63. $\int \frac{1}{x(ax^2+bx^3+cx^4)^{3/2}}\,dx$

$$-\frac{-\frac{\frac{15b(7b^2-12ac)(b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7\ldots}{}}{a(b^2-4ac)}$$

$$\frac{2(-2ac+b^2+bcx)}{ax^2(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\frac{15b(7b^2-12ac)(b^2-4ac)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-\ldots}{}}{a(b^2-4ac)}$$

$$\frac{2(-2ac+b^2+bcx)}{ax^2(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]
) + (-1/3*((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^4) - (-1/2*(
b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) - (-(((105*b^4 -
 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (15*b*
(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x
^2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))/(6*a))/(a*(b^2 - 4*a*c))
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{15\left(-\frac{7x^2b^2\left(-\frac{92}{7}c^2x^2-\frac{106}{7}bcx+b^2\right)a^{\frac{3}{2}}}{72}+\frac{7x\left(-\frac{128}{7}c^3x^3-\frac{244}{7}bc^2x^2+\frac{86}{7}b^2cx+b^3\right)a^{\frac{5}{2}}}{180}+\frac{\left(-16c^2x^2-7bcx-b^2\right)a^{\frac{7}{2}}}{45}+\frac{4a^{\frac{9}{2}}c}{45}+\left(-\frac{7b \ldots}{}\right.\right.}{\sqrt{cx^2+bx+a}\,a^{\frac{9}{2}}(4ac-b^2)x \ldots}$
default	$-\frac{(cx^2+bx+a)\left(-512a^{\frac{7}{2}}c^3x^4+920a^{\frac{5}{2}}b^2c^2x^4-210a^{\frac{3}{2}}b^4cx^4-976a^{\frac{7}{2}}bc^2x^3+1060a^{\frac{5}{2}}b^3cx^3-210a^{\frac{3}{2}}b^5x^3+720\ln\left(\frac{2a+bx+2\sqrt{\ldots}}{}\right.\right.}{\ldots}$
risch	$-\frac{(cx^2+bx+a)(-40acx^2+57b^2x^2-22abx+8a^2)}{24a^4x^2\sqrt{x^2(cx^2+bx+a)}}+\frac{\left(\frac{2b^4cx}{a^4(4ac-b^2)\sqrt{cx^2+bx+a}}+\frac{b^5}{a^4(4ac-b^2)\sqrt{cx^2+bx+a}}+\frac{4c^3x}{a^2(4ac-b^2)\sqrt{cx \ldots}}\right.}{\ldots}$

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15*(-7/72*x^2*b^2*(-92/7*c^2*x^2-106/7*b*c*x+b^2)*a^(3/2)+7/180*x*(-128/7
*c^3*x^3-244/7*b*c^2*x^2+86/7*b^2*c*x+b^3)*a^(5/2)+1/45*(-16*c^2*x^2-7*b*c
*x-b^2)*a^(7/2)+4/45*a^(9/2)*c+(-7/24*b^3*(c*x+b)*a^(1/2)+(c*x^2+b*x+a)^(1
/2)*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(a*c-7/
12*b^2)*(a*c-1/4*b^2))*x^3*b)/(c*x^2+b*x+a)^(1/2)/a^(9/2)/(4*a*c-b^2)/x^3
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \left[-\frac{15\,((7\,b^5c-40\,ab^3c^2+48\,a^2bc^3)x^6+(7\,b^6-40\,ab^4c+48\,a^2b^2c^2)x^5+(7\,a\right.$$

$$-\frac{15\,((7\,b^5c-40\,ab^3c^2+48\,a^2bc^3)x^6+(7\,b^6-40\,ab^4c+48\,a^2b^2c^2)x^5+(7\,ab^5-40\,a^2b^3c+48\,a^3bc^2)x^4)\sqrt{-a}}{}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^
4*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*s
qrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^
4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 48
8*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^
3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2
)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((
7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*
b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arcta
n(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2
+ a^2*x)) + 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256
*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*
b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(
c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*
c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4)]
```

3.63.6 Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

3.63.7 Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

3.63.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

3.64 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx$

3.64.1 Optimal result

Integrand size = 24, antiderivative size = 343

$$\begin{aligned}\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx &= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,x^3\sqrt{ax^2+bx^3+cx^4}} \\ &- \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2(b^2-4ac)\,x^5} + \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3(b^2-4ac)\,x^4} \\ &- \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4(b^2-4ac)\,x^3} \\ &+ \frac{b(315b^4-1680ab^2c+1808a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{64a^5(b^2-4ac)\,x^2} \\ &- \frac{15(21b^4-56ab^2c+16a^2c^2)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}\end{aligned}$$

output
```
-15/128*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*
x^4+b*x^3+a*x^2)^(1/2))/a^(11/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c
*x^4+b*x^3+a*x^2)^(1/2)-1/4*(-20*a*c+9*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/
(-4*a*c+b^2)/x^5+1/8*b*(-68*a*c+21*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*
a*c+b^2)/x^4-1/32*(240*a^2*c^2-448*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1
/2)/a^4/(-4*a*c+b^2)/x^3+1/64*b*(1808*a^2*c^2-1680*a*b^2*c+315*b^4)*(c*x^4
+b*x^3+a*x^2)^(1/2)/a^5/(-4*a*c+b^2)/x^2
```

3.64.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2 (ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\sqrt{a}(-64a^5c-315b^5x^4(b+cx)-105ab^3x^3(b^2-18bcx-16c^2x^2)+16a^4(b^2$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[a]*(-64*a^5*c - 315*b^5*x^4*(b + c*x) - 105*a*b^3*x^3*(b^2 - 18*b*c*
x - 16*c^2*x^2) + 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 2*a^2*b*x^2*(21*b^
3 + 308*b^2*c*x - 1352*b*c^2*x^2 - 904*c^3*x^3) - 8*a^3*x*(3*b^3 + 26*b^2*
c*x + 98*b*c^2*x^2 - 60*c^3*x^3)) - 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2
*c^2 - 64*a^3*c^3)*x^4*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a +
 x*(b + c*x)])/Sqrt[a]])/(64*a^(11/2)*(-b^2 + 4*a*c)*x^3*Sqrt[x^2*(a + x*(
b + c*x))])
```

3.64.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1971, 27, 1998, 27, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1971}$$

$$\frac{2(-2ac+b^2+bcx)}{ax^3(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{9b^2+8cxb-20ac}{2x^4\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9b^2+8cxb-20ac}{x^4\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax^3(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{\int \frac{3\left(b\left(21b^2-68ac\right)+2c\left(9b^2-20ac\right)x\right)}{2x^3\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{3\int \frac{b\left(21b^2-68ac\right)+2c\left(9b^2-20ac\right)x}{x^3\sqrt{cx^4+bx^3+ax^2}}dx}{8a}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 1998$$

$$\frac{-\frac{3\left(-\frac{\int \frac{105b^4-448acb^2+4c\left(21b^2-68ac\right)xb+240a^2c^2}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)}{8a}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{3\left(-\frac{\int \frac{105b^4-448acb^2+4c\left(21b^2-68ac\right)xb+240a^2c^2}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)}{8a}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 1998$$

$$\frac{-\frac{3\left(-\frac{-\frac{\int \frac{b\left(315b^4-1680acb^2+1808a^2c^2\right)+2c\left(105b^4-448acb^2+240a^2c^2\right)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)}{8a}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 27$$

3.64. $\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

$$-\frac{3\left(-\frac{-\frac{\int \frac{b\left(315b^4-1680acb^2+1808a^2c^2\right)+2c\left(105b^4-448acb^2+240a^2c^2\right)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right.}{8a}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 1998

$$-\frac{3\left(-\frac{-\frac{-\frac{\int \frac{15\left(b^2-4ac\right)\left(21b^4-56acb^2+16a^2c^2\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\left(21b^2\right.}{}\right.}{8a}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 27

$$-\frac{3\left(-\frac{-\frac{-\frac{15\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx}}{2ax^3}}{6a}\right.}{8a}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 1951

$$-\frac{3\left(-\frac{-\frac{-\frac{15\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(240a^2c^2-448ab^2c+\right.}{2}}{6a}\right.}{8a}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 219

3.64. $\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

$$-\frac{3\left(-\frac{-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}-\frac{\frac{15\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b\left(1808a^2c^2-1680ab^2c+315b\cdots\right.}{ax^2}}{4a}}{6a}\right.\cdots}{8a}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4]
) + (-1/4*((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^5) - (3*(-1/
3*(b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^4) - (-1/2*((105*
b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) - (-
((b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
a*x^2)) + (15*(b^2 - 4*a*c)*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*ArcTanh[(x*
(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))
/(6*a)))/(8*a))/(a*(b^2 - 4*a*c))
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{15\left(\frac{2a^{\frac{11}{2}}c}{15}+\frac{7b^3x^3\left(-16c^2x^2-18bcx+b^2\right)a^{\frac{3}{2}}}{32}-\frac{7\left(-\frac{904}{21}c^3x^3-\frac{1352}{21}bc^2x^2+\frac{44}{3}b^2cx+b^3\right)x^2ba^{\frac{5}{2}}}{80}+\frac{x\left(-20c^3x^3+\frac{98}{3}bc^2x^2+\frac{26}{3}b^2cx+\right.}{20}}{8a}$
risch	$-\frac{(cx^2+bx+a)\left(292abcx^3-187b^3x^3-56a^2cx^2+82ab^2x^2-40a^2bx+16a^3\right)}{64a^5x^3\sqrt{x^2(cx^2+bx+a)}}+\frac{\left(\frac{374b^5(2cx+b)}{\left(4ac-b^2\right)\sqrt{cx^2+bx+a}}-\frac{288a^2bc^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{cx^2+bx+}}\right.}{}$
default	$-\frac{(cx^2+bx+a)\left(3616a^{\frac{7}{2}}bc^3x^5-3360a^{\frac{5}{2}}b^3c^2x^5+630a^{\frac{3}{2}}b^5cx^5-960a^{\frac{9}{2}}c^3x^4+5408a^{\frac{7}{2}}b^2c^2x^4-3780a^{\frac{5}{2}}b^4cx^4+630a^{\frac{3}{2}}b^6x^4+9\right.}{}$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15/8/a^(11/2)*(2/15*a^(11/2)*c+7/32*b^3*x^3*(-16*c^2*x^2-18*b*c*x+b^2)*a^
(3/2)-7/80*(-904/21*c^3*x^3-1352/21*b*c^2*x^2+44/3*b^2*c*x+b^3)*x^2*b*a^(5
/2)+1/20*x*(-20*c^3*x^3+98/3*b*c^2*x^2+26/3*b^2*c*x+b^3)*a^(7/2)+(-1/5*b*c
*x-1/3*c^2*x^2-1/30*b^2)*a^(9/2)+(21/32*b^5*(c*x+b)*a^(1/2)+(-ln(2)+ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(a^2*c^2-7/2*a*b^2*c+21/1
6*b^4)*(c*x^2+b*x+a)^(1/2)*(a*c-1/4*b^2))*x^4)/(c*x^2+b*x+a)^(1/2)/x^4/(a*
c-1/4*b^2)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 866, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)^{3/2}} \, dx = \left[\frac{15 \left((21 b^6 c - 140 ab^4 c^2 + 240 a^2 b^2 c^3 - 64 a^3 c^4) x^7 + (21 b^7 - 140 ab^5 c + 24\right.}{} \right.$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7
+ (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6
- 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(a)*log(-(8*a*b*x
^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*
a)*sqrt(a))/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*
c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2
 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 +
2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)
*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 -
 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b
^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a
^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c
^2 - 64*a^4*c^3)*x^5)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x
 + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c -
 (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890
*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*
b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x
^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c
- 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5)]
```

3.64.6 Sympy [F]

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{x^2\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

3.64.7 Maxima [F]

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

3.64.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

input
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

3.65 $\int x^m(ax + bx^3 + cx^5)\ dx$

3.65.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int x^m \left(ax + bx^3 + cx^5\right)\ dx = \frac{ax^{2+m}}{2+m} + \frac{bx^{4+m}}{4+m} + \frac{cx^{6+m}}{6+m}$$

output
```
a*x^(2+m)/(2+m)+b*x^(4+m)/(4+m)+c*x^(6+m)/(6+m)
```

3.65.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \left(ax + bx^3 + cx^5\right)\ dx = x^{2+m}\left(\frac{a}{2+m} + \frac{bx^2}{4+m} + \frac{cx^4}{6+m}\right)$$

input
```
Integrate[x^m*(a*x + b*x^3 + c*x^5),x]
```

output
```
x^(2 + m)*(a/(2 + m) + (b*x^2)/(4 + m) + (c*x^4)/(6 + m))
```

3.65.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(ax+bx^3+cx^5\right) \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^{m+1} \left(a+bx^2+cx^4\right) dx$$

$$\Big\downarrow \; 1433$$

$$\int \left(ax^{m+1}+bx^{m+3}+cx^{m+5}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^{m+2}}{m+2}+\frac{bx^{m+4}}{m+4}+\frac{cx^{m+6}}{m+6}$$

input
```
Int[x^m*(a*x + b*x^3 + c*x^5),x]
```

output
```
(a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6 + m)
```

3.65.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.65.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result	size
norman	$\frac{a\,x^{2} e^{m\ln(x)}}{2+m}+\frac{b\,x^{4} e^{m\ln(x)}}{4+m}+\frac{c\,x^{6} e^{m\ln(x)}}{6+m}$	47
gosper	$\frac{x^{2+m}\left(c\,m^{2} x^{4}+6cm\,x^{4}+b\,m^{2} x^{2}+8c\,x^{4}+8bm\,x^{2}+a\,m^{2}+12b\,x^{2}+10am+24a\right)}{(2+m)(4+m)(6+m)}$	77
risch	$\frac{x^{m}\left(c\,m^{2} x^{4}+6cm\,x^{4}+b\,m^{2} x^{2}+8c\,x^{4}+8bm\,x^{2}+a\,m^{2}+12b\,x^{2}+10am+24a\right)x^{2}}{(6+m)(4+m)(2+m)}$	78
parallelrisch	$\frac{x^{6} x^{m} c\,m^{2}+6x^{6} x^{m} cm+8x^{6} x^{m} c+x^{4} x^{m} b\,m^{2}+8x^{4} x^{m} bm+12x^{4} x^{m} b+x^{2} x^{m} a\,m^{2}+10x^{2} x^{m} am+24x^{2} x^{m} a}{(6+m)(4+m)(2+m)}$	108

input
```
int(x^m*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
a/(2+m)*x^2*exp(m*ln(x))+b/(4+m)*x^4*exp(m*ln(x))+c/(6+m)*x^6*exp(m*ln(x))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.92

$$\int x^m\left(ax+bx^3+cx^5\right)\,dx = \frac{\left((cm^2+6\,cm+8\,c)x^6+(bm^2+8\,bm+12\,b)x^4+(am^2+10\,am+24\,a)x^2\right)x^m}{m^3+12\,m^2+44\,m+48}$$

input
```
integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2 + 10*a*m
+ 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)
```

3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(29) = 58$.

Time = 0.34 (sec) , antiderivative size = 280, normalized size of antiderivative = 7.57

$$\int x^m \left(ax+bx^3+cx^5\right) dx = \begin{cases} -\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x) \\ -\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2} \\ a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4} \\ \frac{am^2x^2x^m}{m^3+12m^2+44m+48} + \frac{10amx^2x^m}{m^3+12m^2+44m+48} + \frac{24ax^2x^m}{m^3+12m^2+44m+48} + \frac{bm^2x^4x^m}{m^3+12m^2+44m+48} + \frac{8bmx^4x^m}{m^3+12m^2+44m+48} + \frac{12bx^4x}{m^3+12m^2+} \end{cases}$$

input
```
integrate(x**m*(c*x**5+b*x**3+a*x),x)
```

output
```
Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(2*x**2) +
 b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 + c*x**4/4, Eq(m, -
2)), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 10*a*m*x**2*x**m/(m*
*3 + 12*m**2 + 44*m + 48) + 24*a*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) +
b*m**2*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12
*m**2 + 44*m + 48) + 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*
x**6*x**m/(m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 +
 44*m + 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^m \left(ax+bx^3+cx^5\right) dx = \frac{cx^{m+6}}{m+6} + \frac{bx^{m+4}}{m+4} + \frac{ax^{m+2}}{m+2}$$

input
```
integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
c*x^(m + 6)/(m + 6) + b*x^(m + 4)/(m + 4) + a*x^(m + 2)/(m + 2)
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(37) = 74.

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int x^m\left(ax+bx^3+cx^5\right)\,dx$$
$$=\frac{cm^2x^6x^m+6\,cmx^6x^m+bm^2x^4x^m+8\,cx^6x^m+8\,bmx^4x^m+am^2x^2x^m+12\,bx^4x^m+10\,amx^2x^m+24\,ax}{m^3+12\,m^2+44\,m+48}$$

input
```
integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
(c*m^2*x^6*x^m + 6*c*m*x^6*x^m + b*m^2*x^4*x^m + 8*c*x^6*x^m + 8*b*m*x^4*x
^m + a*m^2*x^2*x^m + 12*b*x^4*x^m + 10*a*m*x^2*x^m + 24*a*x^2*x^m)/(m^3 +
12*m^2 + 44*m + 48)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.41

$$\int x^m\left(ax+bx^3+cx^5\right)\,dx=x^m\left(\frac{a\,x^2\,\left(m^2+10\,m+24\right)}{m^3+12\,m^2+44\,m+48}+\frac{b\,x^4\,\left(m^2+8\,m+12\right)}{m^3+12\,m^2+44\,m+48}+\frac{c\,x^6\,\left(m^2+6\,m+8\right)}{m^3+12\,m^2+44\,m+48}\right)$$

input
```
int(x^m*(a*x + b*x^3 + c*x^5),x)
```

output
```
x^m*((a*x^2*(10*m + m^2 + 24))/(44*m + 12*m^2 + m^3 + 48) + (b*x^4*(8*m +
m^2 + 12))/(44*m + 12*m^2 + m^3 + 48) + (c*x^6*(6*m + m^2 + 8))/(44*m + 12
*m^2 + m^3 + 48))
```

3.66 $\int x^2(ax + bx^3 + cx^5)\,dx$

3.66.1 Optimal result

Integrand size = 18, antiderivative size = 25

$$\int x^2\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

output
```
1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

3.66.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

input
```
Integrate[x^2*(a*x + b*x^3 + c*x^5),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

3.66.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax + bx^3 + cx^5\right) \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^3 \left(a + bx^2 + cx^4\right) dx$$

$$\Big\downarrow \text{ 1433}$$

$$\int \left(ax^3 + bx^5 + cx^7\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

input
```
Int[x^2*(a*x + b*x^3 + c*x^5),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

3.66.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}+\frac{1}{8}c\,x^{8}$	20
norman	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}+\frac{1}{8}c\,x^{8}$	20
risch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}+\frac{1}{8}c\,x^{8}$	20
parallelrisch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}+\frac{1}{8}c\,x^{8}$	20
gosper	$\frac{x^{4}\left(3c\,x^{4}+4b\,x^{2}+6a\right)}{24}$	22

input
```
int(x^2*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(ax+bx^3+cx^5\right)\,dx=\frac{1}{8}\,cx^8+\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

input
```
integrate(x**2*(c*x**5+b*x**3+a*x),x)
```

output
```
a*x**4/4 + b*x**6/6 + c*x**8/8
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3+cx^5)\,dx=\frac{1}{8}\,cx^8+\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3+cx^5)\,dx=\frac{1}{8}\,cx^8+\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax+bx^3+cx^5)\,dx=\frac{c\,x^8}{8}+\frac{b\,x^6}{6}+\frac{a\,x^4}{4}$$

input
```
int(x^2*(a*x + b*x^3 + c*x^5),x)
```

output
```
(a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

3.67 $\int x(ax+bx^3+cx^5)\,dx$

3.67.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int x\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

output
```
1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

3.67.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

input
```
Integrate[x*(a*x + b*x^3 + c*x^5),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

3.67.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {9, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^3+cx^5\right)\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^2\left(a+bx^2+cx^4\right)dx$$

$$\Big\downarrow \text{ 1433}$$

$$\int \left(ax^2+bx^4+cx^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

input
```
Int[x*(a*x + b*x^3 + c*x^5),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

3.67.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
parallelrisch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
gosper	$\frac{x^{3}\left(15c\,x^{4}+21b\,x^{2}+35a\right)}{105}$	22

input
```
int(x*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\left(ax+bx^3+cx^5\right)\,dx=\frac{1}{7}\,cx^{7}+\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

input
```
integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\left(ax+bx^3+cx^5\right)\,dx=\frac{ax^{3}}{3}+\frac{bx^{5}}{5}+\frac{cx^{7}}{7}$$

input
```
integrate(x*(c*x**5+b*x**3+a*x),x)
```

output
```
a*x**3/3 + b*x**5/5 + c*x**7/7
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3+cx^5)\,dx=\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3+cx^5)\,dx=\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3+cx^5)\,dx=\frac{c\,x^7}{7}+\frac{b\,x^5}{5}+\frac{a\,x^3}{3}$$

input
```
int(x*(a*x + b*x^3 + c*x^5),x)
```

output
```
(a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

3.68 $\int (ax + bx^3 + cx^5) \, dx$

3.68.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int (ax + bx^3 + cx^5) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

output
```
1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

3.68.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ax + bx^3 + cx^5) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

input
```
Integrate[a*x + b*x^3 + c*x^5,x]
```

output
```
(a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

3.68.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^3+cx^5\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

input
```
Int[a*x + b*x^3 + c*x^5,x]
```

output
```
(a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.68.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
risch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
parts	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
gosper	$\frac{x^{2}\left(2c\,x^{4}+3b\,x^{2}+6a\right)}{12}$	22

input
```
int(c*x^5+b*x^3+a*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3+cx^5\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(c*x^5+b*x^3+a*x,x, algorithm="fricas")
```

output
```
1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3+cx^5\right)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

input
```
integrate(c*x**5+b*x**3+a*x,x)
```

output
```
a*x**2/2 + b*x**4/4 + c*x**6/6
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3+cx^5\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(c*x^5+b*x^3+a*x,x, algorithm="maxima")
```

output
```
1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3+cx^5\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(c*x^5+b*x^3+a*x,x, algorithm="giac")
```

output
```
1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3+cx^5\right)\,dx=\frac{c\,x^6}{6}+\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

input
```
int(a*x + b*x^3 + c*x^5,x)
```

output
```
(a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

3.69 $\int \frac{ax+bx^3+cx^5}{x} dx$

3.69.1 Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{ax+bx^3+cx^5}{x} dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

output
```
a*x+1/3*b*x^3+1/5*c*x^5
```

3.69.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3+cx^5}{x} dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)/x,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

3.69.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {9, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax + bx^3 + cx^5}{x} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \left(a + bx^2 + cx^4\right) dx$$
$$\Big\downarrow \; 2009$$
$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
Int[(a*x + b*x^3 + c*x^5)/x,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

3.69.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
norman	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
risch	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
parallelrisch	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
parts	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
gosper	$\frac{x\left(3c\,x^{4}+5b\,x^{2}+15a\right)}{15}$	20

input
```
int((c*x^5+b*x^3+a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x^3+1/5*c*x^5
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax+bx^3+cx^5}{x}\,dx = \frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3+ax$$

input
```
integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{ax+bx^3+cx^5}{x}\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

input
```
integrate((c*x**5+b*x**3+a*x)/x,x)
```

output
```
a*x + b*x**3/3 + c*x**5/5
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax + bx^3 + cx^5}{x}\, dx = \frac{1}{5}\, cx^5 + \frac{1}{3}\, bx^3 + ax$$

input
```
integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax + bx^3 + cx^5}{x}\, dx = \frac{1}{5}\, cx^5 + \frac{1}{3}\, bx^3 + ax$$

input
```
integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax + bx^3 + cx^5}{x}\, dx = \frac{c\, x^5}{5} + \frac{b\, x^3}{3} + a\, x$$

input
```
int((a*x + b*x^3 + c*x^5)/x,x)
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

3.70 $\int \frac{ax+bx^3+cx^5}{x^2} dx$

3.70.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{ax+bx^3+cx^5}{x^2} dx = \frac{bx^2}{2} + \frac{cx^4}{4} + a\log(x)$$

output
```
1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

3.70.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3+cx^5}{x^2} dx = \frac{bx^2}{2} + \frac{cx^4}{4} + a\log(x)$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]
```

output
```
(b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

3.70.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3+cx^5}{x^2}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{a+bx^2+cx^4}{x}dx$$

$$\Big\downarrow \ 1433$$

$$\int \left(\frac{a}{x}+bx+cx^3\right)dx$$

$$\Big\downarrow \ 2009$$

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

input
```
Int[(a*x + b*x^3 + c*x^5)/x^2,x]
```

output
```
(b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

3.70.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.70.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
parallelrisch	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
norman	$\frac{\frac{1}{2}b\,x^{3}+\frac{1}{4}c\,x^{5}}{x}+a\ln\left(x\right)$	23
risch	$\frac{c\,x^{4}}{4}+\frac{b\,x^{2}}{2}+\frac{b^{2}}{4c}+a\ln\left(x\right)$	26

input
```
int((c*x^5+b*x^3+a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax+bx^3+cx^5}{x^2}\,dx=\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+a\log\left(x\right)$$

input
```
integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="fricas")
```

output
```
1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax+bx^3+cx^5}{x^2}\,dx=a\log\left(x\right)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

input
```
integrate((c*x**5+b*x**3+a*x)/x**2,x)
```

output
```
a*log(x) + b*x**2/2 + c*x**4/4
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax + bx^3 + cx^5}{x^2} \, dx = \frac{1}{4} cx^4 + \frac{1}{2} bx^2 + a \log (x)$$

input
```
integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="maxima")
```

output
```
1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{ax + bx^3 + cx^5}{x^2} \, dx = \frac{1}{4} cx^4 + \frac{1}{2} bx^2 + \frac{1}{2} a \log \left(x^2\right)$$

input
```
integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="giac")
```

output
```
1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{ax + bx^3 + cx^5}{x^2} \, dx = \frac{b\, x^2}{2} + \frac{c\, x^4}{4} + a\, \ln (x)$$

input
```
int((a*x + b*x^3 + c*x^5)/x^2,x)
```

output
```
(b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

3.71 $\int \frac{ax+bx^3+cx^5}{x^3}\,dx$

3.71.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx=-\frac{a}{x}+bx+\frac{cx^3}{3}$$

output
```
-a/x+b*x+1/3*c*x^3
```

3.71.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx=-\frac{a}{x}+bx+\frac{cx^3}{3}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]
```

output
```
-(a/x) + b*x + (c*x^3)/3
```

3.71.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+bx^3+cx^5}{x^3}dx$$

$$\Big\downarrow 9$$

$$\int \frac{a+bx^2+cx^4}{x^2}dx$$

$$\Big\downarrow 1433$$

$$\int \left(\frac{a}{x^2}+b+cx^2\right)dx$$

$$\Big\downarrow 2009$$

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

input
```
Int[(a*x + b*x^3 + c*x^5)/x^3,x]
```

output
```
-(a/x) + b*x + (c*x^3)/3
```

3.71.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
risch	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
norman	$\frac{b\,x^{3}-ax+\frac{1}{3}c\,x^{5}}{x^{2}}$	21
parallelrisch	$\frac{c\,x^{4}+3b\,x^{2}-3a}{3x}$	21
gosper	$-\frac{-c\,x^{4}-3b\,x^{2}+3a}{3x}$	22

input
```
int((c*x^5+b*x^3+a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*x+1/3*c*x^3
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx=\frac{cx^4+3\,bx^2-3\,a}{3\,x}$$

input
```
integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="fricas")
```

output
```
1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{ax+bx^3+cx^5}{x^3}\,dx=-\frac{a}{x}+bx+\frac{cx^3}{3}$$

input
```
integrate((c*x**5+b*x**3+a*x)/x**3,x)
```

output
```
-a/x + b*x + c*x**3/3
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{ax + bx^3 + cx^5}{x^3} \, dx = \frac{1}{3} \, cx^3 + bx - \frac{a}{x}$$

input
```
integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + b*x - a/x
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{ax + bx^3 + cx^5}{x^3} \, dx = \frac{1}{3} \, cx^3 + bx - \frac{a}{x}$$

input
```
integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="giac")
```

output
```
1/3*c*x^3 + b*x - a/x
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{ax + bx^3 + cx^5}{x^3} \, dx = b\,x - \frac{a}{x} + \frac{c\,x^3}{3}$$

input
```
int((a*x + b*x^3 + c*x^5)/x^3,x)
```

output
```
b*x - a/x + (c*x^3)/3
```

3.72 $\int x^m(ax+bx^3+cx^5)^2\,dx$

3.72.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int x^m\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^{3+m}}{3+m}+\frac{2abx^{5+m}}{5+m}+\frac{(b^2+2ac)\,x^{7+m}}{7+m}+\frac{2bcx^{9+m}}{9+m}+\frac{c^2x^{11+m}}{11+m}$$

output
```
a^2*x^(3+m)/(3+m)+2*a*b*x^(5+m)/(5+m)+(2*a*c+b^2)*x^(7+m)/(7+m)+2*b*c*x^(9
+m)/(9+m)+c^2*x^(11+m)/(11+m)
```

3.72.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int x^m\left(ax+bx^3+cx^5\right)^2\,dx=x^{3+m}\left(\frac{a^2}{3+m}+\frac{2abx^2}{5+m}+\frac{(b^2+2ac)\,x^4}{7+m}+\frac{2bcx^6}{9+m}+\frac{c^2x^8}{11+m}\right)$$

input
```
Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
x^(3 + m)*(a^2/(3 + m) + (2*a*b*x^2)/(5 + m) + ((b^2 + 2*a*c)*x^4)/(7 + m)
 + (2*b*c*x^6)/(9 + m) + (c^2*x^8)/(11 + m))
```

3.72.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(ax+bx^3+cx^5\right)^2 dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{m+2}\left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{ 1433}$$

$$\int \left(a^2x^{m+2}+x^{m+6}\left(2ac+b^2\right)+2abx^{m+4}+2bcx^{m+8}+c^2x^{m+10}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^{m+3}}{m+3}+\frac{x^{m+7}\left(2ac+b^2\right)}{m+7}+\frac{2abx^{m+5}}{m+5}+\frac{2bcx^{m+9}}{m+9}+\frac{c^2x^{m+11}}{m+11}$$

input
```
Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a*c)*x^(7
+ m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 + m))/(11 + m)
```

3.72.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(76) = 152.

Time = 0.14 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.95

method	result
gosper	$\frac{x^{3+m}(c^2m^4x^8+24c^2m^3x^8+2bc\,m^4x^6+206c^2m^2x^8+52bc\,m^3x^6+744m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+472bc\,m^2x^6+945c^2x^8+56a}{}$
risch	$\frac{x^m(c^2m^4x^8+24c^2m^3x^8+2bc\,m^4x^6+206c^2m^2x^8+52bc\,m^3x^6+744m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+472bc\,m^2x^6+945c^2x^8+56ac}{}$
parallelrisch	$\frac{24x^{11}x^mc^2m^3+206x^{11}x^mc^2m^2+744x^{11}x^mc^2m+x^7x^mb^2m^4+28x^7x^mb^2m^3+2310x^9x^mbc+4158x^5x^mab+374x^3x^ma^2m^2+1}{}$

input
```
int(x^m*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
x^(3+m)/(3+m)/(5+m)/(7+m)/(9+m)/(11+m)*(c^2*m^4*x^8+24*c^2*m^3*x^8+2*b*c*m
^4*x^6+206*c^2*m^2*x^8+52*b*c*m^3*x^6+744*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*
x^4+472*b*c*m^2*x^6+945*c^2*x^8+56*a*c*m^3*x^4+28*b^2*m^3*x^4+1772*b*c*m*x
^6+2*a*b*m^4*x^2+548*a*c*m^2*x^4+274*b^2*m^2*x^4+2310*b*c*x^6+60*a*b*m^3*x
^2+2184*a*c*m*x^4+1092*b^2*m*x^4+a^2*m^4+640*a*b*m^2*x^2+2970*a*c*x^4+1485
*b^2*x^4+32*a^2*m^3+2820*a*b*m*x^2+374*a^2*m^2+4158*a*b*x^2+1888*a^2*m+346
5*a^2)
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(76) = 152.

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.17

$$\int x^m\left(ax+bx^3+cx^5\right)^2\,dx$$
$$=\frac{((c^2m^4+24\,c^2m^3+206\,c^2m^2+744\,c^2m+945\,c^2)x^{11}+2\,(bcm^4+26\,bcm^3+236\,bcm^2+886\,bcm+1155}{}$$

input
```
integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
((c^2*m^4 + 24*c^2*m^3 + 206*c^2*m^2 + 744*c^2*m + 945*c^2)*x^11 + 2*(b*c*
m^4 + 26*b*c*m^3 + 236*b*c*m^2 + 886*b*c*m + 1155*b*c)*x^9 + ((b^2 + 2*a*c
)*m^4 + 28*(b^2 + 2*a*c)*m^3 + 274*(b^2 + 2*a*c)*m^2 + 1485*b^2 + 2970*a*c
 + 1092*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 30*a*b*m^3 + 320*a*b*m^2 + 141
0*a*b*m + 2079*a*b)*x^5 + (a^2*m^4 + 32*a^2*m^3 + 374*a^2*m^2 + 1888*a^2*m
 + 3465*a^2)*x^3)*x^m/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

3.72.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. $2(66) = 132$.

Time = 0.65 (sec) , antiderivative size = 1377, normalized size of antiderivative = 18.12

$$\int x^m (ax + bx^3 + cx^5)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Piecewise((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) -
b*c/x**2 + c**2*log(x), Eq(m, -11)), (-a**2/(6*x**6) - a*b/(2*x**4) - a*c/
x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2, Eq(m, -9)), (-a**2/(4*x
**4) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x**4/4, Eq(
m, -7)), (-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x**
4/2 + c**2*x**6/6, Eq(m, -5)), (a**2*log(x) + a*b*x**2 + a*c*x**4/2 + b**2
*x**4/4 + b*c*x**6/3 + c**2*x**8/8, Eq(m, -3)), (a**2*m**4*x**3*x**m/(m**5
 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 32*a**2*m**3*x**3*x*
*m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 374*a**2*m**
2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 188
8*a**2*m*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395
) + 3465*a**2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m +
10395) + 2*a*b*m**4*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 912
9*m + 10395) + 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**
2 + 9129*m + 10395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 +
3010*m**2 + 9129*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 35*m**4 + 470*m
**3 + 3010*m**2 + 9129*m + 10395) + 4158*a*b*x**5*x**m/(m**5 + 35*m**4 + 4
70*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a*c*m**4*x**7*x**m/(m**5 + 35*m*
*4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5
+ 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*...
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int x^m(ax+bx^3+cx^5)^2\,dx = \frac{c^2x^{m+11}}{m+11}+\frac{2\,bcx^{m+9}}{m+9}+\frac{b^2x^{m+7}}{m+7}+\frac{2\,acx^{m+7}}{m+7}+\frac{2\,abx^{m+5}}{m+5}+\frac{a^2x^{m+3}}{m+3}$$

input
```
integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
c^2*x^(m + 11)/(m + 11) + 2*b*c*x^(m + 9)/(m + 9) + b^2*x^(m + 7)/(m + 7)
+ 2*a*c*x^(m + 7)/(m + 7) + 2*a*b*x^(m + 5)/(m + 5) + a^2*x^(m + 3)/(m + 3
)
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(76) = 152.

Time = 0.29 (sec) , antiderivative size = 399, normalized size of antiderivative = 5.25

$$\int x^m(ax+bx^3+cx^5)^2\,dx$$
$$= \frac{c^2m^4x^{11}x^m+24\,c^2m^3x^{11}x^m+2\,bcm^4x^9x^m+206\,c^2m^2x^{11}x^m+52\,bcm^3x^9x^m+744\,c^2mx^{11}x^m+b^2m^4x^7x}{}$$

input
```
integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
(c^2*m^4*x^11*x^m + 24*c^2*m^3*x^11*x^m + 2*b*c*m^4*x^9*x^m + 206*c^2*m^2*
x^11*x^m + 52*b*c*m^3*x^9*x^m + 744*c^2*m*x^11*x^m + b^2*m^4*x^7*x^m + 2*a
*c*m^4*x^7*x^m + 472*b*c*m^2*x^9*x^m + 945*c^2*x^11*x^m + 28*b^2*m^3*x^7*x
^m + 56*a*c*m^3*x^7*x^m + 1772*b*c*m*x^9*x^m + 2*a*b*m^4*x^5*x^m + 274*b^2
*m^2*x^7*x^m + 548*a*c*m^2*x^7*x^m + 2310*b*c*x^9*x^m + 60*a*b*m^3*x^5*x^m
 + 1092*b^2*m*x^7*x^m + 2184*a*c*m*x^7*x^m + a^2*m^4*x^3*x^m + 640*a*b*m^2
*x^5*x^m + 1485*b^2*x^7*x^m + 2970*a*c*x^7*x^m + 32*a^2*m^3*x^3*x^m + 2820
*a*b*m*x^5*x^m + 374*a^2*m^2*x^3*x^m + 4158*a*b*x^5*x^m + 1888*a^2*m*x^3*x
^m + 3465*a^2*x^3*x^m)/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395
)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.57

$$\int x^m \left(ax+bx^3+cx^5\right)^2 \, dx = \frac{a^2 x^m x^3 \left(m^4+32 m^3+374 m^2+1888 m+3465\right)}{m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395} + \frac{c^2 x^m x^{11} \left(m^4+24 m^3+206 m^2+744 m+945\right)}{m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395} + \frac{x^m x^7 \left(b^2+2 a c\right) \left(m^4+28 m^3+274 m^2+1092 m+1485\right)}{m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395} + \frac{2 a b x^m x^5 \left(m^4+30 m^3+320 m^2+1410 m+2079\right)}{m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395} + \frac{2 b c x^m x^9 \left(m^4+26 m^3+236 m^2+886 m+1155\right)}{m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395}$$

input
```
int(x^m*(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
(a^2*x^m*x^3*(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))/(9129*m + 3010*m^2
+ 470*m^3 + 35*m^4 + m^5 + 10395) + (c^2*x^m*x^11*(744*m + 206*m^2 + 24*m^
3 + m^4 + 945))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (x^
m*x^7*(2*a*c + b^2)*(1092*m + 274*m^2 + 28*m^3 + m^4 + 1485))/(9129*m + 30
10*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (2*a*b*x^m*x^5*(1410*m + 320*m^
2 + 30*m^3 + m^4 + 2079))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10
395) + (2*b*c*x^m*x^9*(886*m + 236*m^2 + 26*m^3 + m^4 + 1155))/(9129*m + 3
010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395)
```

3.73 $\int x^2(ax+bx^3+cx^5)^2\,dx$

3.73.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int x^2\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

3.73.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^2\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

input
```
Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c
^2*x^13)/13
```

3.73.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax+bx^3+cx^5\right)^2 \, dx$$
$$\Big\downarrow \; 9$$
$$\int x^4 \left(a+bx^2+cx^4\right)^2 dx$$
$$\Big\downarrow \; 1433$$
$$\int \left(a^2x^4+x^8 \left(2ac+b^2\right)+2abx^6+2bcx^{10}+c^2x^{12}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{a^2x^5}{5}+\frac{1}{9}x^9 \left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

input
```
Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c
^2*x^13)/13
```

3.73.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^5}{5}+\frac{2ab\,x^7}{7}+\frac{(2ac+b^2)x^9}{9}+\frac{2bc\,x^{11}}{11}+\frac{c^2x^{13}}{13}$	45
norman	$\frac{c^2x^{13}}{13}+\frac{2bc\,x^{11}}{11}+\left(\frac{2ac}{9}+\frac{b^2}{9}\right)x^9+\frac{2ab\,x^7}{7}+\frac{a^2x^5}{5}$	46
risch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{2}{11}bc\,x^{11}+\frac{1}{13}c^2x^{13}$	47
parallelrisch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{2}{11}bc\,x^{11}+\frac{1}{13}c^2x^{13}$	47
gosper	$\frac{x^5(3465c^2x^8+8190bc\,x^6+10010ac\,x^4+5005b^2x^4+12870ab\,x^2+9009a^2)}{45045}$	49

input
```
int(x^2*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2\left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\,\left(b^2+2\,ac\right)x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*
a^2*x^5
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^2(ax+bx^3+cx^5)^2\,dx=\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{2bcx^{11}}{11}+\frac{c^2x^{13}}{13}+x^9\cdot\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

input
```
integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)
```

output
```
a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**9*(2*a*c/
9 + b**2/9)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2(ax+bx^3+cx^5)^2\,dx=\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\,(b^2+2\,ac)x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*
a^2*x^5
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2(ax+bx^3+cx^5)^2\,dx=\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\,b^2x^9+\frac{2}{9}\,acx^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a*b*x^7 +
1/5*a^2*x^5
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x^2 \left(ax + bx^3 + cx^5\right)^2 \, dx = x^9 \left(\frac{b^2}{9} + \frac{2\,a\,c}{9}\right) + \frac{a^2\,x^5}{5} + \frac{c^2\,x^{13}}{13} + \frac{2\,a\,b\,x^7}{7} + \frac{2\,b\,c\,x^{11}}{11}$$

input
```
int(x^2*(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
x^9*((2*a*c)/9 + b^2/9) + (a^2*x^5)/5 + (c^2*x^13)/13 + (2*a*b*x^7)/7 + (2
*b*c*x^11)/11
```

3.74 $\int x(ax+bx^3+cx^5)^2\,dx$

3.74.1 Optimal result

Integrand size = 18, antiderivative size = 54

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{120}x^4\left(30a^2+40abx^2+15\left(b^2+2ac\right)x^4+24bcx^6+10c^2x^8\right)$$

input
```
Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 10*c^2*x^8
))/120
```

3.74.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {9, 1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^3\left(a+bx^2+cx^4\right)^2dx$$
$$\Big\downarrow \text{ 1434}$$
$$\frac{1}{2}\int x^2\left(cx^4+bx^2+a\right)^2dx^2$$
$$\Big\downarrow \text{ 1140}$$
$$\frac{1}{2}\int \left(c^2x^{10}+2bcx^8+\left(b^2+2ac\right)x^6+2abx^4+a^2x^2\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{a^2x^4}{2}+\frac{1}{4}x^8\left(2ac+b^2\right)+\frac{2}{3}abx^6+\frac{2}{5}bcx^{10}+\frac{c^2x^{12}}{6}\right)$$

input
```
Int[x*(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((a^2*x^4)/2 + (2*a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/4 + (2*b*c*x^10)/5 + (c
^2*x^12)/6)/2
```

3.74.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^4}{4}+\frac{ab\,x^6}{3}+\frac{(2ac+b^2)x^8}{8}+\frac{bc\,x^{10}}{5}+\frac{c^2x^{12}}{12}$	45
norman	$\frac{c^2x^{12}}{12}+\frac{bc\,x^{10}}{5}+\left(\frac{ac}{4}+\frac{b^2}{8}\right)x^8+\frac{ab\,x^6}{3}+\frac{a^2x^4}{4}$	46
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{1}{5}bc\,x^{10}+\frac{1}{12}c^2x^{12}$	47
parallelrisch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{1}{5}bc\,x^{10}+\frac{1}{12}c^2x^{12}$	47
gosper	$\frac{x^4(10c^2x^8+24bc\,x^6+30ac\,x^4+15b^2x^4+40ab\,x^2+30a^2)}{120}$	49

input
```
int(x*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x\left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,(b^2+2\,ac)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a
^2*x^4
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(ax+bx^3+cx^5)^2\,dx=\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{bcx^{10}}{5}+\frac{c^2x^{12}}{12}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

input
```
integrate(x*(c*x**5+b*x**3+a*x)**2,x)
```

output
```
a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*c/4 + b**
2/8)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(ax+bx^3+cx^5)^2\,dx=\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,(b^2+2\,ac)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a
^2*x^4
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(ax+bx^3+cx^5)^2\,dx=\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,b^2x^8+\frac{1}{4}\,acx^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*b*x^6 + 1
/4*a^2*x^4
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x(ax+bx^3+cx^5)^2\,dx = x^8\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{a^2\,x^4}{4}+\frac{c^2\,x^{12}}{12}+\frac{a\,b\,x^6}{3}+\frac{b\,c\,x^{10}}{5}$$

input
```
int(x*(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
x^8*((a*c)/4 + b^2/8) + (a^2*x^4)/4 + (c^2*x^12)/12 + (a*b*x^6)/3 + (b*c*x
^10)/5
```

3.75 $\int (ax+bx^3+cx^5)^2\,dx$

3.75.1 Optimal result

Integrand size = 16, antiderivative size = 54

$$\int (ax+bx^3+cx^5)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

3.75.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (ax+bx^3+cx^5)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2
*x^11)/11
```

3.75.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1949, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^3+cx^5\right)^2 dx$$

$$\Big\downarrow \text{1949}$$

$$\int x^2\left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(a^2x^2+x^6\left(2ac+b^2\right)+2abx^4+2bcx^8+c^2x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

input
```
Int[(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2
*x^11)/11
```

3.75.3.1 Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^3}{3}+\frac{2ab\,x^5}{5}+\frac{(2ac+b^2)x^7}{7}+\frac{2bc\,x^9}{9}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{2bc\,x^9}{9}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\frac{2ab\,x^5}{5}+\frac{a^2x^3}{3}$	46
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{2}{9}bc\,x^9+\frac{1}{11}c^2x^{11}$	47
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{2}{9}bc\,x^9+\frac{1}{11}c^2x^{11}$	47
gosper	$\frac{x^3(315c^2x^8+770bc\,x^6+990ac\,x^4+495b^2x^4+1386ab\,x^2+1155a^2)}{3465}$	49

input
```
int((c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,\left(b^2+2\,ac\right)x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^
2*x^3
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{2bcx^9}{9}+\frac{c^2x^{11}}{11}+x^7\cdot\left(\frac{2ac}{7}+\frac{b^2}{7}\right)$$

input
```
integrate((c*x**5+b*x**3+a*x)**2,x)
```

output
```
a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7
+ b**2/7)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,b^2x^7+\frac{1}{3}\,a^2x^3+\frac{2}{35}\,\left(5\,cx^7+7\,bx^5\right)a$$

input
```
integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 1/3*a^2*x^3 + 2/35*(5*c*x^7 +
7*b*x^5)*a
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/
3*a^2*x^3
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \left(ax+bx^3+cx^5\right)^2\,dx=x^7\,\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^3}{3}+\frac{c^2\,x^{11}}{11}+\frac{2\,a\,b\,x^5}{5}+\frac{2\,b\,c\,x^9}{9}$$

input
```
int((a*x + b*x^3 + c*x^5)^2,x)
```

output
```
x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2
*b*c*x^9)/9
```

3.76 $\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx$

3.76.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = \frac{a^2x^2}{2} + \frac{1}{2}abx^4 + \frac{1}{6}\left(b^2+2ac\right)x^6 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

output
```
1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

3.76.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = \frac{1}{60}x^2\left(30a^2+30abx^2+10\left(b^2+2ac\right)x^4+15bcx^6+6c^2x^8\right)$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]
```

output
```
(x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8)
)/60
```

3.76.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 1432, 1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x(a+bx^2+cx^4)^2\,dx$$

$$\Big\downarrow \; 1432$$

$$\frac{1}{2}\int (cx^4+bx^2+a)^2\,dx^2$$

$$\Big\downarrow \; 1085$$

$$\frac{1}{2}\int \left(c^2x^8+2bcx^6+b^2\left(\frac{2ac}{b^2}+1\right)x^4+2abx^2+a^2\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(a^2x^2+\frac{1}{3}x^6(2ac+b^2)+abx^4+\frac{1}{2}bcx^8+\frac{c^2x^{10}}{5}\right)$$

input
```
Int[(a*x + b*x^3 + c*x^5)^2/x,x]
```

output
```
(a^2*x^2 + a*b*x^4 + ((b^2 + 2*a*c)*x^6)/3 + (b*c*x^8)/2 + (c^2*x^10)/5)/2
```

3.76.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^2}{2}+\frac{ab\,x^4}{2}+\frac{(2ac+b^2)x^6}{6}+\frac{bc\,x^8}{4}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{bc\,x^8}{4}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}$	46
risch	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	47
parallelrisch	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	47
gosper	$\frac{x^2(6c^2x^8+15bc\,x^6+20ac\,x^4+10b^2x^4+30ab\,x^2+30a^2)}{60}$	49

input
```
int((c*x^5+b*x^3+a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx=\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="fricas")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^
2*x^2
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = \frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

input
```
integrate((c*x**5+b*x**3+a*x)**2/x,x)
```

output
```
a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2
/6)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = \frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,(b^2+2\,ac)x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

input
```
integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="maxima")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^
2*x^2
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = \frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

input
```
integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="giac")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/
2*a^2*x^2
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx = x^6\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^{10}}{10}+\frac{a\,b\,x^4}{2}+\frac{b\,c\,x^8}{4}$$

input
```
int((a*x + b*x^3 + c*x^5)^2/x,x)
```

output
```
x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x
^8)/4
```

3.77 $\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx$

3.77.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

3.77.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

3.77.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \; 1403$$

$$\int \left(a^2+b^2x^4\left(\frac{2ac}{b^2}+1\right)+2abx^2+2bcx^6+c^2x^8\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Int[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

3.77.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.77.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$a^2x+\frac{2ab\,x^3}{3}+\frac{(2ac+b^2)x^5}{5}+\frac{2bc\,x^7}{7}+\frac{c^2x^9}{9}$	42
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	44
parallelrisch	$a^2x+\frac{2}{3}ab\,x^3+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	44
gosper	$\frac{x(35c^2x^8+90bc\,x^6+126ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$	47
norman	$\frac{a^2x^2+\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^6+\frac{c^2x^{10}}{9}+\frac{2ab\,x^4}{3}+\frac{2bc\,x^8}{7}}{x}$	49

input
```
int((c*x^5+b*x^3+a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,(b^2+2\,ac)x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\cdot\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

input
```
integrate((c*x**5+b*x**3+a*x)**2/x**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = \frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,(b^2+2\,ac)x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = \frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*
x
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx = a^2\,x + x^5\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{c^2\,x^9}{9}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^7}{7}$$

input
```
int((a*x + b*x^3 + c*x^5)^2/x^2,x)
```

output
```
a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7
)/7
```

3.78 $\int \frac{x^8}{ax+bx^3+cx^5}\,dx$

3.78.1 Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx=-\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

output
```
-1/2*b*x^2/c^2+1/4*x^4/c+1/4*(-a*c+b^2)*ln(c*x^4+b*x^2+a)/c^3+1/2*b*(-3*a*
c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx=\frac{cx^2(-2b+cx^2)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

input
```
Integrate[x^8/(a*x + b*x^3 + c*x^5),x]
```

output
```
(c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3
)
```

3.78.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow 9$$

$$\int \frac{x^7}{a+bx^2+cx^4}dx$$

$$\Big\downarrow 1434$$

$$\frac{1}{2}\int \frac{x^6}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow 1143$$

$$\frac{1}{2}\int \left(\frac{x^2}{c}+\frac{(b^2-ac)\,x^2+ab}{c^2\,(cx^4+bx^2+a)}-\frac{b}{c^2}\right)dx^2$$

$$\Big\downarrow 2009$$

$$\frac{1}{2}\left(\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{2c^3}-\frac{bx^2}{c^2}+\frac{x^4}{2c}\right)$$

input
```
Int[x^8/(a*x + b*x^3 + c*x^5),x]
```

output
```
(-((b*x^2)/c^2) + x^4/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x^
4])/(2*c^3))/2
```

3.78.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

method	result
default	$-\frac{-\frac{1}{2} c\,x^{4}+b\,x^{2}}{2 c^{2}}+\frac{\frac{\left(-a c +b^{2}\right) \ln \left(c\,x^{4}+b\,x^{2}+a \right)}{2 c}+\frac{2 \left(a b -\frac{\left(-a c +b^{2}\right) b}{2 c}\right) \arctan \left(\frac{2 c\,x^{2}+b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{2 c^{2}}$
risch	$\frac{x^{4}}{4 c}-\frac{b\,x^{2}}{2 c^{2}}+\frac{b^{2}}{4 c^{3}}-\frac{\ln \left(\left(\left(12 a^{2} b\,c^{2}-7 a\,b^{3} c +b^{5}+\sqrt{-b^{2} \left(4 a c -b^{2}\right) \left(3 a c -b^{2}\right)^{2}}\, b \right) x^{2}+2 \sqrt{-b^{2} \left(4 a c -b^{2}\right) \left(3 a c -b^{2}\right)^{2}}\, a \right) a^{2}}{c \left(4 a c -b^{2}\right)}+\frac{5 \ln \left(\left(\left(12\right.\right.\right.}{}$

input
```
int(x^8/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/2*c*x^4+b*x^2)+1/2/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^4+b*x^2+a)+2*
(a*b-1/2*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2)))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.13

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \left[\frac{(b^2c^2-4\,ac^3)x^4-2\,(b^3c-4\,abc^2)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+}{4\,(b^2c^3-4\,ac^4)}\right.$$

input
```
integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 +
b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)
*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4
 - 2*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

3.78.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(92) = 184.

Time = 1.75 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.91

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = -\frac{bx^2}{2c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-}{4c^{}}\right.}{3abc-b^3}\right.$$
$$+\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right.$$
$$+\frac{x^4}{4c}$$

input
```
integrate(x**8/(c*x**5+b*x**3+a*x),x)
```

output
```
-b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
 b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*
(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b*
*2)/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3
*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)
)*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c -
 b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*s
qrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(
4*c**3)))/(3*a*b*c - b**3)) + x**4/(4*c)
```

3.78.7 Maxima [F]

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \int \frac{x^8}{cx^5+bx^3+ax}\,dx$$

input
```
integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/4*(c*x^4 - 2*b*x^2)/c^2 - integrate(-((b^2 - a*c)*x^3 + a*b*x)/(c*x^4 +
b*x^2 + a), x)/c^2
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \frac{cx^4-2\,bx^2}{4\,c^2} + \frac{(b^2-ac)\log\left(cx^4+bx^2+a\right)}{4\,c^3} - \frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1
/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c^3)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 842, normalized size of antiderivative = 8.42

$$\int \frac{x^8}{ax+bx^3+cx^5}\,dx = \frac{x^4}{4\,c} - \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)} - \frac{b\,x^2}{2\,c^2}$$

$$+ \frac{b\,\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{b\,\left(3\,a\,c-b^2\right)\,\left(\frac{8\,a^2\,c^4-8\,a\,b^2\,c^3}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)}{8\,c^3\,\sqrt{4\,a\,c-b^2}}-\frac{a\,b\,\left(3\,a\,c-b^2\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{c\,\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}}{a}-x^2\left(\frac{b\left(\frac{6\,b^3 \cdots}{\cdots}\right.}{\cdots}\right.\right)}{\cdots}\right)}{\cdots}$$

input
```
int(x^8/(a*x + b*x^3 + c*x^5),x)
```

output
```
x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(
16*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*((
(b*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*
c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(8*c^3*(4*a*c - b^2)^(1/2)) -
(a*b*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2
)*(16*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4
*b*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c -
b^2))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2
- 10*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((
b^5 + 2*a^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b
*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*
a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/
(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*
a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*
b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (
a*b^4 + a^3*c^2 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c -
 b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3
*a*c - b^2))/(2*c^3*(4*a*c - b^2)^(1/2))
```

3.79 $\int \frac{x^7}{ax+bx^3+cx^5}\,dx$

3.79.1 Optimal result

Integrand size = 20, antiderivative size = 203

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx=-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-b*x/c^2+1/3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx=-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^7/(a*x + b*x^3 + c*x^5),x]
```

output
```
-((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3
- 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.79.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {9, 1442, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{ax+bx^3+cx^5}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^6}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{x^3}{3c}-\frac{\int \frac{3x^2(bx^2+a)}{cx^4+bx^2+a}dx}{3c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3}{3c}-\frac{\int \frac{x^2(bx^2+a)}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{x^3}{3c}-\frac{\frac{bx}{c}-\frac{\int \frac{(b^2-ac)x^2+ab}{cx^4+bx^2+a}dx}{c}}{c}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{x^3}{3c}-\frac{\frac{bx}{c}-\frac{\frac{1}{2}\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3}{3c}-\frac{\frac{bx}{c}-\frac{\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}$$

input
```
Int[x^7/(a*x + b*x^3 + c*x^5),x]
```

output
```
x^3/(3*c) - ((b*x)/c - (((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c
```

3.79.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result
risch	$\frac{x^3}{3c} - \frac{bx}{c^2} + \frac{\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\left(-ac+b^2\right)\_R^2+ab\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}}{2c^2}$
default	$-\frac{-\frac{1}{3}c\,x^3+bx}{c^2} + \frac{\frac{\left(-ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}-3abc+b^3\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\left(-ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+3abc-b^3\right)\sqrt{}}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\right.}}}{c}$

input
```
int(x^7/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c-b*x/c^2+1/2/c^2*sum(((-a*c+b^2)*_R^2+a*b)/(2*_R^3*c+_R*b)*ln(x-_
R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(167) = 334$.

Time = 0.31 (sec) , antiderivative size = 1564, normalized size of antiderivative = 7.70

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2
*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^
3*b^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3
*b*c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4
*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^
5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*
c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*
c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*
c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 -
 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*
c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*...
```

3.79.6 Sympy [A] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{ax+bx^3+cx^5}\,dx = -\frac{bx}{c^2}$$
$$+\text{RootSum}\left(t^4\cdot\left(256a^2c^7-128ab^2c^6+16b^4c^5\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+a^5,\left(t\mapsto\right.\right.$$
$$+\frac{x^3}{3c}$$

input
```
integrate(x**7/(c*x**5+b*x**3+a*x),x)
```

output
```
-b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
 + _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a
**5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8
*_t**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5
*c - 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

3.79.7 Maxima [F]

$$\int \frac{x^7}{ax + bx^3 + cx^5} \, dx = \int \frac{x^7}{cx^5 + bx^3 + ax} \, dx$$

input
```
integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^
2 + a), x)/c^2
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(167) = 334$.

Time = 0.61 (sec) , antiderivative size = 2457, normalized size of antiderivative = 12.10

$$\int \frac{x^7}{ax + bx^3 + cx^5} \, dx = \text{Too large to display}$$

input
```
integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^
2*c^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*
b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2...
```

3.79.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 4127, normalized size of antiderivative = 20.33

$$\int \frac{x^7}{ax + bx^3 + cx^5} \, dx = \text{Too large to display}$$

input
```
int(x^7/(a*x + b*x^3 + c*x^5),x)
```

output
```
x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 1
6*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b
^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-
(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c
^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2
) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c
^5 - 16*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25
*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c
^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2
*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4...
```

3.79. $\int \frac{x^7}{ax+bx^3+cx^5}\,dx$

3.80 $\int \frac{x^6}{ax+bx^3+cx^5}\,dx$

3.80.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx=\frac{x^2}{2c}-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

output
```
1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx=\frac{2cx^2+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

input
```
Integrate[x^6/(a*x + b*x^3 + c*x^5),x]
```

output
```
(2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

3.80.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^5}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^4}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1143}$$

$$\frac{1}{2}\int \left(\frac{1}{c}-\frac{bx^2+a}{c\,(cx^4+bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{x^2}{c}\right)$$

input
```
Int[x^6/(a*x + b*x^3 + c*x^5),x]
```

output
```
(x^2/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(2*c^2))/2
```

3.80.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.80.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^2}{2c}+\frac{-\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$
risch	$\frac{x^2}{2c}-\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^2+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{c(4ac-b^2)}+\frac{\ln\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(\right.\right.}{}$

input
```
int(x^6/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c+1/2/c*(-1/2*b/c*ln(c*x^4+b*x^2+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)x^2-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)\log\left(cx^4\right.}{4\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
 b*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)
, 1/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*
x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(71) = 142$.

Time = 1.11 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \left(-\frac{b}{4c^2}\right.$$
$$\left.-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2\right.}{4c^2\cdot(4ac-}}{2ac-b^2}\right.$$
$$+\left(-\frac{b}{4c^2}\right.$$
$$\left.+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2\right.}{4c^2\cdot(4ac-}}{2ac-b^2}\right.$$
$$+\frac{x^2}{2c}$$

input
```
integrate(x**6/(c*x**5+b*x**3+a*x),x)
```

output
```
(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))
*log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b
**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2
)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2)
+ sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (
-a*b - 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*
(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

3.80.7 Maxima [F]

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \int \frac{x^6}{cx^5+bx^3+ax}\,dx$$

input
```
integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
1/2*x^2/c - integrate((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \frac{x^2}{2\,c} - \frac{b\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2
*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 655, normalized size of antiderivative = 8.09

$$\int \frac{x^6}{ax+bx^3+cx^5}\,dx = \frac{x^2}{2\,c} + \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}$$

$$+ \frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\,\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}-x^2\,\left(\frac{\frac{\left(2\,a\,c-b^2\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\,a\,b \cdots\right.}{16\,a\,c^3-4\,b^2\,c^2 \cdots}\right.}{8\,c^2\,\sqrt{4\,a\,c-b^2}}}{a} \cdots\right.\right.}{\cdots}\right)}{\cdots}$$

input
```
int(x^6/(a*x + b*x^3 + c*x^5),x)
```

output
```
x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^
2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a
*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c
^2)))/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^
3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^
2)))/a + (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b
^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3
- a*b*c)/c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b
^2)^(1/2))) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^
3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*
a*c - b^2)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a
^2*c^2 - 4*a*b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

3.81 $\int \frac{x^5}{ax+bx^3+cx^5} dx$

3.81.1 Optimal result

Integrand size = 20, antiderivative size = 179

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*
a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = \frac{x}{c} - \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^5/(a*x + b*x^3 + c*x^5),x]
```

output
```
x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.81.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^4}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{x}{c}-\frac{\int \frac{bx^2+a}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}$$

input
```
Int[x^5/(a*x + b*x^3 + c*x^5),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c
```

3.81.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.81.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\_R^2b-a\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{x}{c}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

input
```
int(x^5/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/2/c*sum((-_R^2*b-a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b
+a))
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(143) = 286$.

Time = 0.27 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.92

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = -\frac{\sqrt{\frac{1}{2}}c\sqrt{-\frac{b^3-3\,abc+(b^2c^3-4\,ac^4)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{b^2c^6-4\,ac^7}}}{b^2c^3-4\,ac^4}}\log\left(-2\,(ab^2-a^2c)x+\sqrt{\frac{1}{2}}\left(b^4-5\,ab^2c+4\,a^2c^2-(b^3c^3-4\,a\right.\right.}{}$$

input
```
integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b
^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*
c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*
c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2
*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*
sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sq
rt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^
3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*s...
```

3.81.6 Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3\right.\right.\right.$$
$$+\frac{x}{c}$$

input
```
integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48
*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t*
*3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b*
*4)/(a**2*c - a*b**2)))) + x/c
```

3.81.7 Maxima [F]

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx=\int \frac{x^5}{cx^5+bx^3+ax}\,dx$$

input
```
integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. $2(143) = 286$.

Time = 0.56 (sec) , antiderivative size = 2109, normalized size of antiderivative = 11.78

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
x/c - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c
^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*s...
```

3.81.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 3026, normalized size of antiderivative = 16.91

$$\int \frac{x^5}{ax + bx^3 + cx^5} \, dx = \text{Too large to display}$$

input
```
int(x^5/(a*x + b*x^3 + c*x^5),x)
```

output
```
x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)
*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c
)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) -
 (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c +
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^
4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 ...
```

3.82 $\int \frac{x^4}{ax+bx^3+cx^5}\,dx$

3.82.1 Optimal result

Integrand size = 20, antiderivative size = 63

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

output
```
1/4*ln(c*x^4+b*x^2+a)/c+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-
4*a*c+b^2)^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx=\frac{-\frac{2b\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^2+cx^4\right)}{4c}$$

input
```
Integrate[x^4/(a*x + b*x^3 + c*x^5),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + b*x^2 + c*x^4])/(4*c)
```

3.82.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {9, 1434, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^3}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^2}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{c}+\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)$$

input
```
Int[x^4/(a*x + b*x^3 + c*x^5),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(2*c))/2
```

3.82.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.82.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4c}-\frac{b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2c\sqrt{4ac-b^{2}}}$
risch	$\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{2}+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,a\right)a}{4ac-b^{2}}-\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{2}+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,a\right)b^{2}}{4c\left(4ac-b^{2}\right)}+$

input
```
int(x^4/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)}, \frac{2\sqrt{-b^2+4\,ac}b\,\mathrm{ar}}{}\right.$$

input
```
integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*
x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4
 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c
*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*
x^2 + a))/(b^2*c - 4*a*c^2)]
```

3.82.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 0.50 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

input
```
integrate(x**4/(c*x**5+b*x**3+a*x),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c
*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-
b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c
 + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c
 + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

3.82.7 Maxima [F]

$$\int \frac{x^4}{ax+bx^3+cx^5}\,dx = \int \frac{x^4}{cx^5+bx^3+ax}\,dx$$

input
```
integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^5 + b*x^3 + a*x), x)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax + bx^3 + cx^5}\,dx = -\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

input
```
integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
-1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1
/4*log(c*x^4 + b*x^2 + a)/c
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \frac{x^4}{ax + bx^3 + cx^5}\,dx = \frac{4\,a\,c\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c} - \frac{b^2\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^4/(a*x + b*x^3 + c*x^5),x)
```

output
```
(4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
 c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/
(4*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

3.83 $\int \frac{x^3}{ax+bx^3+cx^5}\,dx$

3.83.1 Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^
(1/2)/(-4*a*c+b^2)^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[x^3/(a*x + b*x^3 + c*x^5),x]
```

output
```
((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

3.83.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^2}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \ 1450$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \ 218$$

$$\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[x^3/(a*x + b*x^3 + c*x^5),x]
```

output
```
((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((1 + b/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.83.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

3.83.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}\right)}{2}$	41
default	$4c\left(\frac{\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(-b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,c\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)$	149

input
```
int(x^3/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(115) = 230$.

Time = 0.28 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.73

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$

input

```
integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

3.83. $\int \frac{x^3}{ax+bx^3+cx^5}\,dx$

output
```
1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
 - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/
sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) -
 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*
c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2
)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
 - 1/2*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^
2*c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^
2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x
) + 1/2*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b
^2*c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*
c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) +
 x)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{ax + bx^3 + cx^5}\,dx$$
$$= \operatorname{RootSum}\left(t^4 \cdot \left(256a^2c^3 - 128ab^2c^2 + 16b^4c\right) + t^2\left(-16abc + 4b^3\right) + a, \left(t \mapsto t\log\left(64t^3ac^2 - 16t^3b^2c - 2tb\right.\right.\right.$$

input
```
integrate(x**3/(c*x**5+b*x**3+a*x),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a
*b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c -
2*_t*b + x)))
```

3.83.7 Maxima [F]

$$\int \frac{x^3}{ax + bx^3 + cx^5}\,dx = \int \frac{x^3}{cx^5 + bx^3 + ax}\,dx$$

input
```
integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^5 + b*x^3 + a*x), x)
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(115) = 230$.

Time = 0.64 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.35

$$\int \frac{x^3}{ax + bx^3 + cx^5}\,dx$$
$$= \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc+\sqrt{b^2-4\,ac}c}ac + 2\,\sqrt{2}\sqrt{b^2 -}\right.}{2\,(b^4 - 8\,ab^2c - 2\,b^3c + 16\,a^2}$$
$$+ \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc-\sqrt{b^2-4\,ac}c}ac + 2\,\sqrt{2}\sqrt{b}\right.}{2\,(b^4 - 8\,ab^2c - 2\,b^3c + 16}$$

input
```
integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 -
8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
+ 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2
- 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4
- 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)
)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.77

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = -2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^2-2\,b^2\,c)+\frac{x\,(8\,b^3\,c^2-32\,a\,b\,c^3)\,\left(b^3+\sqrt{-(4\,a\,c-b^2)^3}-4\,a\,b\,c\right)}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c)}\right)\sqrt{-\frac{b^3+\sqrt{-(4\,a\,c-b^2)^3}-4\,a\,b\,c}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c)}}}{a\,c}\right)\sqrt{-\cdots}$$
$$-2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^2-2\,b^2\,c)-\frac{x\,(8\,b^3\,c^2-32\,a\,b\,c^3)\,\left(\sqrt{-(4\,a\,c-b^2)^3}-b^3+4\,a\,b\,c\right)}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c)}\right)\sqrt{\frac{\sqrt{-(4\,a\,c-b^2)^3}-b^3+4\,a\,b\,c}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c)}}}{a\,c}\right)\sqrt{\frac{\sqrt{-\cdots}}{8\,(\cdots}}$$

input
```
int(x^3/(a*x + b*x^3 + c*x^5),x)
```

output
```
- 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(
4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*
(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*
b^2*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c
) - (x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)
)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3
 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a
*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))
)^(1/2)
```

3.84 $\int \frac{x^2}{ax+bx^3+cx^5} dx$

3.84.1 Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{x^2}{ax+bx^3+cx^5} dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{ax+bx^3+cx^5} dx = \frac{\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[x^2/(a*x + b*x^3 + c*x^5),x]
```

output
```
ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

3.84.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \; 1432$$

$$\frac{1}{2}\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \; 1083$$

$$-\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)$$

$$\Big\downarrow \; 219$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a*x + b*x^3 + c*x^5),x]
```

output
```
-(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

3.84.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

3.84.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	36
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^2-2a\right)}{2\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^2+2a\right)}{2\sqrt{-4ac+b^2}}$	70

input
```
int(x^2/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.58

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},\ -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(
-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

3.84.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.64

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

input
```
integrate(x**2/(c*x**5+b*x**3+a*x),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2
*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2
+ (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)
)/2
```

3.84.7 Maxima [F]

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx = \int \frac{x^2}{cx^5+bx^3+ax}\,dx$$

input
```
integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^5 + b*x^3 + a*x), x)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx=\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{ax+bx^3+cx^5}\,dx=\frac{\mathrm{atan}\left(\frac{2\,a\,c\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^2/(a*x + b*x^3 + c*x^5),x)
```

output
```
atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

3.85 $\int \frac{x}{ax+bx^3+cx^5}\,dx$

3.85.1 Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x}{ax+bx^3+cx^5}\,dx=\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4
*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{x}{ax+bx^3+cx^5}\,dx=\frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[x/(a*x + b*x^3 + c*x^5),x]
```

output
```
(Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

3.85.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1406}$$

$$\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[x/(a*x + b*x^3 + c*x^5),x]
```

output
```
(Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]])
```

3.85.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

3.85.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{4\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{4\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)$	117

input
```
int(x/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(114) = 228$.

Time = 0.28 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.09

$$\begin{aligned}\int \frac{x}{ax+bx^3+cx^5}\,dx = & -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx\right. \\ & \left.+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) \\ & +\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx\right. \\ & \left.-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) \\ & -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx\right. \\ & \left.+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) \\ & +\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx\right. \\ & \left.-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)\end{aligned}$$

input
```
integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^
2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqr
t(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)
)/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c -
(a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/
sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a
*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sq
rt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-
(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*
sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4
*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2
*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c)))
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{x}{ax + bx^3 + cx^5}\,dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(256a^3c^2 - 128a^2b^2c + 16ab^4\right) + t^2\left(-16abc + 4b^3\right) + c, \left(t \mapsto t\log\left(x + \frac{32t^3a^2bc - 8t^3ab^3}{c}\right.\right.\right.$$

input
```
integrate(x/(c*x**5+b*x**3+a*x),x)
```

output
```
RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a
*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b
**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.85.7 Maxima [F]

$$\int \frac{x}{ax + bx^3 + cx^5}\,dx = \int \frac{x}{cx^5 + bx^3 + ax}\,dx$$

input
```
integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^5 + b*x^3 + a*x), x)
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. 2(114) = 228.

Time = 0.59 (sec) , antiderivative size = 1024, normalized size of antiderivative = 6.83

$$\int \frac{x}{ax+bx^3+cx^5}\,dx$$
$$= \frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c-2\,b^4c+16\,\sqrt{2}\sqrt{}\right.}{}$$
$$+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^3c+2\,b^4c+16\,\sqrt{2}\right.}{}$$

input
```
integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2...
```

3.85.9 Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{x}{ax + bx^3 + cx^5}\,dx =$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i} + b\,x\,\sqrt{-64\,a^3\,c^3 + 48\,a^2\,b^2\,c^2 - 12\,a\,b^4\,c + b^6}\,1\mathrm{i} + a^2\,c^2 \cdots}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}} + 64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b \cdots}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}} \cdots}\right.$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i} - b\,x\,\sqrt{-64\,a^3\,c^3 + 48\,a^2\,b^2\,c^2 - 12\,a\,b^4\,c + b^6}\,1\mathrm{i} + a^2\,c^2\,x \cdots}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}} + 64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}} - \cdots}\right.$$

input
```
int(x/(a*x + b*x^3 + c*x^5),x)
```

output
```
- atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 -
64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))
^(1/2) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(
b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8
*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*
b^2*c*x*8i)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^
3*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a
*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b
^2*c))^(1/2)*2i
```

3.86 $\int \frac{1}{ax+bx^3+cx^5}\,dx$

3.86.1 Optimal result

Integrand size = 16, antiderivative size = 69

$$\int \frac{1}{ax+bx^3+cx^5}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

output
```
ln(x)/a-1/4*ln(c*x^4+b*x^2+a)/a+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/a/(-4*a*c+b^2)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{1}{ax+bx^3+cx^5}\,dx =\frac{4\sqrt{b^2-4ac}\log(x)-\left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+\right.}{4a\sqrt{b^2-4ac}}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)^(-1),x]
```

output
```
(4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

3.86.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1949, 1434, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax+bx^3+cx^5}dx$$

$$\Big\downarrow \text{1949}$$

$$\int \frac{1}{x(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}+\frac{\log(x^2)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^4+bx^2+a}dx^2+\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[(a*x + b*x^3 + c*x^5)^(-1),x]
```

output
```
(Log[x^2]/a - (-((b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^2 + c*x^4]/2)/a)/2
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

3.86.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$	65
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ac-3b^2\right)\_R+5c\right)x^2-ab\_R+2b\right)\right)}{2}$	77

input
```
int(1/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/2/a*(1/2*ln(c*x^4+b*x^2+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2)))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{ax+bx^3+cx^5}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)+4\,(b^2-4\,ac)\log}{4\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*
x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4
 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
2 - 4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2
*c)]
```

3.86.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 8.64 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{ax+bx^3+cx^5}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)} - \frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)} - \frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right) + \frac{\log(x)}{a}$$

input
```
integrate(1/(c*x**5+b*x**3+a*x),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b
*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)
) + (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*
a**2*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(
b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c
)) + log(x)/a
```

3.86.7 Maxima [F]

$$\int \frac{1}{ax+bx^3+cx^5}\,dx = \int \frac{1}{cx^5+bx^3+ax}\,dx$$

input
```
integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
-integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1}{ax+bx^3+cx^5}\,dx = -\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^4+bx^2+a\right)}{4\,a} + \frac{\log\left(x^2\right)}{2\,a}$$

input
```
integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
-1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1
/4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

3.86.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 1014, normalized size of antiderivative = 14.70

$$\int \frac{1}{ax+bx^3+cx^5}\,dx = \frac{\ln(x)}{a} + \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}$$
$$+ \frac{b\,\mathrm{atan}\left(\frac{16\,a^3\,x^2\left(\frac{\left(3\,b^3-8\,a\,b\,c\right)\left(\frac{\left(8\,a\,c-2\,b^2\right)^2\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{4\,\left(4\,a\,b^2-16\,a^2\,c\right)^2}-\frac{b^2\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{16\,a^2\,\left(4\,a\,c-b^2\right)}+\frac{b^2\,(\cdots}{16\cdots}\right.}{8\,a^3\,c^2\,\left(25\,a\,c-6\,b^2\right)}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
int(1/(a*x + b*x^3 + c*x^5),x)
```

output
```
log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c
)) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3
 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/
(4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(
8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(
12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*
a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b
^2*c)*((b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(
12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(
4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b
*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c
)*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))
*(4*a*c - b^2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)
*(((8*a*c - 2*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 -
 16*a^2*c)))/(4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(
8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(
8*a*c - 2*b^2))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*
c - 6*b^2)) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2
)/(16*a^2*(4*a*c - b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 -
 16*a^2*c)^2*(4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*...
```

3.86. $\int \frac{1}{ax+bx^3+cx^5}\,dx$

3.87 $\int \frac{1}{x(ax+bx^3+cx^5)}\,dx$

3.87.1 Optimal result

Integrand size = 20, antiderivative size = 174

$$\int \frac{1}{x(ax+bx^3+cx^5)}\,dx = -\frac{1}{ax} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*
(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)
^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(ax+bx^3+cx^5)}\,dx = -\frac{\frac{2}{x} + \frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

input
```
Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

output
```
-1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/a
```

3.87.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1443, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}$$

input
```
Int[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

output
```
-(1/(a*x)) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

3.87.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.87.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

method	result
default	$\frac{4c\left(\frac{\left(b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(12a^2bc^2-7ab^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22a^4b^2c+3b^4a^3\right)\_R^4+(25a\right.\right.\right.}{2}$

input
```
int(1/x/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
4/a*c*(1/8*(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1
/8*(-b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/a/
x
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. $2(137) = 274$.

Time = 0.27 (sec) , antiderivative size = 1116, normalized size of antiderivative = 6.41

$$\int \frac{1}{x\,(ax+bx^3+cx^5)}\,dx = -\frac{\sqrt{\frac{1}{2}}ax\sqrt{-\frac{b^3-3\,abc+(a^3b^2-4\,a^4c)\sqrt{\frac{b^4-2\,ab^2c+a^2c^2}{a^6b^2-4\,a^7c}}}{a^3b^2-4\,a^4c}}\log\left(-2\,(b^2c^2-ac^3)x+\sqrt{\frac{1}{2}}\left(b^5-5\,ab^3c+4\,a^2bc^2-(a^3b^4-\right.\right.}{}$$

input
```
integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b
^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 -
6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7
*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-
(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1
/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c
 + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2
- 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a
^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
- sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c
^2 - a*c^3)*x - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6...
```

3.87.6 Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx$$
$$= \mathrm{RootSum}\left(t^4\cdot\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64\right.\right.\right.$$
$$-\frac{1}{ax}$$

input
```
integrate(1/x/(c*x**5+b*x**3+a*x),x)
```

output
```
RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48
*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t
**3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c*
*2 + 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

3.87.7 Maxima [F]

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx=\int \frac{1}{(cx^5+bx^3+ax)x}\,dx$$

input
```
integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
-integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. $2(137) = 274$.

Time = 0.73 (sec) , antiderivative size = 1839, normalized size of antiderivative = 10.57

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
-1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 -
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*
b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 +
 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32
*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*
arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3...
```

3.87.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 2997, normalized size of antiderivative = 17.22

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a*x + b*x^3 + c*x^5)),x)
```

output
```
- atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(3
2*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3
- 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*
b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))...
```

3.88 $\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx$

3.88.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx = -\frac{1}{2ax^2}-\frac{\left(b^2-2ac\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}$$

output
```
-1/2/a/x^2-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/2*(-2*a*c+b^2)*arctan
h((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx = \frac{-\frac{2a}{x^2}-4b\log(x)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

output
```
((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^
2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^
2)
```

3.88.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {9, 1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax+bx^3+cx^5)}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^3 (a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^4 (cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2}\left(\frac{\int -\frac{cx^2+b}{x^2(cx^4+bx^2+a)}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{cx^2+b}{x^2(cx^4+bx^2+a)}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(-\frac{\int \left(\frac{b}{ax^2}+\frac{-b^2-cx^2b+ac}{a(cx^4+bx^2+a)}\right)dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx^2+cx^4)}{2a}+\frac{b\log(x^2)}{a}}{a}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

output
```
(-(1/(a*x^2)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^2])/a - (b*Log[a + b*x^2 + c*x^4])/(2*a))
/a)/2
```

3.88.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{2a\,x^{2}}-\frac{b\ln(x)}{a^{2}}-\frac{-\frac{b\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}+\frac{2\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{1}{2a\,x^{2}}-\frac{b\ln(x)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(-4abc+b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(10a^{3}c-3a^{2}b^{2}\right)\_R^{2}-4\_Rabc+2c^{2}\right)x^{2}-a\right.\right.}{2}$

input
```
int(1/x^2/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2-b*ln(x)/a^2-1/2/a^2*(-1/2*b*ln(c*x^4+b*x^2+a)+2*(a*c-1/2*b^2)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int\frac{1}{x^{2}\left(ax+bx^{3}+cx^{5}\right)}\,dx$$
$$=\left[-\frac{(b^{2}-2\,ac)\sqrt{b^{2}-4\,ac}x^{2}\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-(b^{3}-4\,abc)x^{2}\log\left(cx^{4}+bx^{2}+a\right)+}{4\,(a^{2}b^{2}-4\,a^{3}c)x^{2}}\right.$$
$$-\frac{2\,(b^{2}-2\,ac)\sqrt{-b^{2}+4\,ac}x^{2}\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-(b^{3}-4\,abc)x^{2}\log\left(cx^{4}+bx^{2}+a\right)+4\,(b^{3}-}{4\,(a^{2}b^{2}-4\,a^{3}c)x^{2}}$$

input
```
integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

output
```
[-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^
2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 -
 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*
b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2
+ 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
3 - 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2
*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**5+b*x**3+a*x),x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)} \, dx = \int \frac{1}{(cx^5 + bx^3 + ax)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

output
```
-b*log(x)/a^2 + integrate((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^2 + a), x
)/a^2 - 1/2/(a*x^2)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)} \, dx = \frac{b \log (cx^4 + bx^2 + a)}{4 a^2} - \frac{b \log (x^2)}{2 a^2} + \frac{(b^2 - 2 ac) \arctan \left(\frac{2 cx^2 + b}{\sqrt{-b^2 + 4 ac}}\right)}{2 \sqrt{-b^2 + 4 ac} a^2} + \frac{bx^2 - a}{2 a^2 x^2}$$

input
```
integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

output
```
1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*
arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b
*x^2 - a)/(a^2*x^2)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 2033, normalized size of antiderivative = 22.84

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a*x + b*x^3 + c*x^5)),x)
```

output
```
(atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a
*b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^
3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c -
 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - (
(((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40
*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a
*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a
*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^
2))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))
)/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a
*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b
*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b
^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4
*a^2*b^2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*
a*c - b^2)^(3/2)) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*
a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*
a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(
4*a^2*(4*a*c - b^2)^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3...
```

3.89 $\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx$

3.89.1 Optimal result

Integrand size = 20, antiderivative size = 166

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \frac{(b^2-3ac)\,x^2}{c^2\,(b^2-4ac)} - \frac{bx^4}{2c\,(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} - \frac{b\log(a+bx^2+cx^4)}{2c^3}$$

output
```
(-3*a*c+b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*b*x^4/c/(-4*a*c+b^2)+1/2*x^6*(b*x^2+
2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/2*b*ln(c*x^4+b*x^2+a)/c
^3
```

3.89.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \frac{cx^2 + \frac{-b^4x^2-ab^2(b-4cx^2)+a^2c(3b-2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} - b\log(a+bx^2+cx^4)}{2c^3}$$

input
```
Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(c*x^2 + (-(b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b
 + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + b*x^2 +
c*x^4])/(2*c^3)
```

3.89.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {9, 1434, 1164, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^9}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^8}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{2}\left(\frac{x^6(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{2x^4(bx^2+3a)}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^6(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2\int \frac{x^4(bx^2+3a)}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{x^6(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2\int \left(\frac{bx^2}{c}-\frac{b^2-3ac}{c^2}+\frac{b(b^2-4ac)x^2+a(b^2-3ac)}{c^2(cx^4+bx^2+a)}\right)dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x^6(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2\left(\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx^2+cx^4)}{2c^3}-\frac{x^2(b^2-3ac)}{c^2}+\frac{bx^4}{2c}\right.}{b^2-4ac}\right.$$

input
```
Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(-(((b^2 - 3
*a*c)*x^2)/c^2) + (b*x^4)/(2*c) + ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c
)*Log[a + b*x^2 + c*x^4])/(2*c^3)))/(b^2 - 4*a*c))/2
```

3.89.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{x^{2}}{2c^{2}}-\frac{\frac{-\frac{\left(2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)x^{2}}{c\left(4ac-b^{2}\right)}+\frac{ba\left(3ac-b^{2}\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(4abc-b^{3}\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{c}+\frac{4\left(3c\,a^{2}-b^{2}a-\frac{\left(4abc-b^{3}\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{4ac-b^{2}}}{2c^{2}}$	209
risch	Expression too large to display	1217

input
```
int(x^11/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c^2-1/2/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x^2+b*a/c*(
3*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c
*ln(c*x^4+b*x^2+a)+2*(3*c*a^2-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(156) = 312$.

Time = 0.29 (sec) , antiderivative size = 868, normalized size of antiderivative = 5.23

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \left[\frac{(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^6 - ab^5 + 7\,a^2b^3c - 12\,a^3bc^2 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^4 - (b^6 - 9\,ab^4c + 2}{}\right.$$

input
```
integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b^3*c - 12*
a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a*b^4*c +
26*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4
*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*
sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*
c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2
*b^2*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3
*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 +
 16*a^2*b*c^5)*x^2), 1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5
 + 7*a^2*b^3*c - 12*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 -
 (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - 2*(a*b^4 - 6*a^2*b^
2*c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c
 + 6*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 -
8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2
)]
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{\left(ax+bx^3+cx^5\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.89.7 Maxima [F]

$$\int \frac{x^{11}}{\left(ax+bx^3+cx^5\right)^2}\,dx=\int \frac{x^{11}}{\left(cx^5+bx^3+ax\right)^2}\,dx$$

input
```
integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*b^3 - 3*a^2*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^2)/(a*b^2*c^3 -
4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*x^2
/c^2 + 2*integrate(-((b^3 - 4*a*b*c)*x^3 + (a*b^2 - 3*a^2*c)*x)/(c*x^4 + b
*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{\left(ax+bx^3+cx^5\right)^2}\,dx=\frac{\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{x^2}{2\,c^2}+\frac{b^3x^4-4\,abcx^4-2\,a^2cx^2-a^2b}{2\,\left(cx^4+bx^2+a\right)\left(b^2c^2-4\,ac^3\right)}-\frac{b\log\left(cx^4+bx^2+a\right)}{2\,c^3}$$

input
```
integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b
^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/2*x^2/c^2 + 1/2*(b^3*x^4 - 4*a*b
*c*x^4 - 2*a^2*c*x^2 - a^2*b)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) -
1/2*b*log(c*x^4 + b*x^2 + a)/c^3
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1473, normalized size of antiderivative = 8.87

$$\int \frac{x^{11}}{(ax + bx^3 + cx^5)^2} dx = \text{Too large to display}$$

input
```
int(x^11/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
((a*(b^3 - 3*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(b^4 + 2*a^2*c^2 - 4*a*b^2
*c))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + x^2/(2*c^2) + (l
og(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/
(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (atan(((4*a*c
^5*(4*a*c - b^2)^3 - b^2*c^4*(4*a*c - b^2)^3)*(((((16*a*b)/c + (8*a*c^2*(b
^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(64*a^3*c^6 - b^6*c^3 +
12*a*b^4*c^4 - 48*a^2*b^2*c^5))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*c^3*(4*a
*c - b^2)^(3/2)) + (4*a*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3
+ 48*a^2*b^3*c^2 - 12*a*b^5*c))/(c*(4*a*c - b^2)^(3/2)*(64*a^3*c^6 - b^6*c
^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) - x^2*(((((4*(6*
a^2*c^5 + 3*b^4*c^3 - 14*a*b^2*c^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 -
 8*a*b*c^7)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5
- b^2*c^4)*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2)) + ((2*b^3*c^6 - 8*a*b
*c^7)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 -
 12*a*b^5*c))/(c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) + (b*((4*(b^
5 + 3*a^2*b*c^2 - 5*a*b^3*c))/(4*a*c^5 - b^2*c^4) + (((4*(6*a^2*c^5 + 3*b^
4*c^3 - 14*a*b^2*c^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 - 8*a*b*c^7)*(b
^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5 - b^2*c^4)*...
```

3.90 $\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$

3.90.1 Optimal result

Integrand size = 20, antiderivative size = 331

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = \frac{(3b^2-10ac)\,x}{2c^2\,(b^2-4ac)} - \frac{bx^3}{2c\,(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3-13abc+\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*(-10*a*c+3*b^2)*x/c^2/(-4*a*c+b^2)-1/2*b*x^3/c/(-4*a*c+b^2)+1/2*x^5*(b
*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(-20*a^2*c^2+19*a*b^2*c-3*b^4)/(-4
*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*
c+(20*a^2*c^2-19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{4\sqrt{c}x - \frac{2\sqrt{c}x(2a^2c-b^3x^2-ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4+19ab^2c-20a^2c^2+3b^3\sqrt{b^2-4ac}-13abc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}(3b}{}}{4c^{5/2}}$$

input
```
Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^
2 + 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[
b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
))/(4*c^(5/2))
```

3.90.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {9, 1440, 1602, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{x^8}{(a+bx^2+cx^4)^2}dx$$
$$\Big\downarrow \ 1440$$
$$\frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^4(3bx^2+10a)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \; 1602$$

$$\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^3}{c}-\frac{\int \frac{3x^2\left(\left(3b^2-10ac\right)x^2+3ab\right)}{cx^4+bx^2+a}dx}{3c}}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^3}{c}-\frac{\int \frac{x^2\left(\left(3b^2-10ac\right)x^2+3ab\right)}{cx^4+bx^2+a}dx}{c}}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1602$$

$$\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^3}{c}-\frac{\frac{x\left(3b^2-10ac\right)}{c}-\frac{\int \frac{b\left(3b^2-13ac\right)x^2+a\left(3b^2-10ac\right)}{cx^4+bx^2+a}dx}{c}}{c}}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^3}{c}-\frac{\frac{x\left(3b^2-10ac\right)}{c}-\frac{\frac{1}{2}\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}}{c}}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^3}{c}-\frac{\frac{x\left(3b^2-10ac\right)}{c}-\frac{\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}}{2\left(b^2-4ac\right)}$$

input
```
Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x^3)/c - (
((3*b^2 - 10*a*c)*x)/c - (((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^
2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^3 - 13*a*b*
c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/c)/c)/(2*(b^2 - 4*a*c))
```

3.90. $\int \frac{x^{10}}{\left(ax+bx^3+cx^5\right)^2}\,dx$

3.90.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.90. $\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$

3.90.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.53

method	result
risch	$\frac{x}{c^2}+\frac{\frac{b(3ac-b^2)x^3}{8ac-2b^2}+\frac{a(2ac-b^2)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{b(13ac-3b^2)\_R^2}{4ac-b^2}-\frac{a(10ac-3b^2)}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4c^2}$
default	$\frac{x}{c^2}-\frac{-\frac{b(3ac-b^2)x^3}{2(4ac-b^2)}-\frac{a(2ac-b^2)x}{2(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(13\sqrt{-4ac+b^2}\,abc-3\sqrt{-4ac+b^2}\,b^3-20a^2c^2+19a\,b^2c-3b^4\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{(13}{}\right.}{4ac-b^2}}{c^2}$

input
```
int(x^10/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*a*(2*a*c-b^2)/(4*a*c-b^2)*x)/
c^2/(c*x^4+b*x^2+a)+1/4/c^2*sum((-b*(13*a*c-3*b^2)/(4*a*c-b^2)*_R^2-a*(10*
a*c-3*b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. 2(285) = 570.

Time = 0.48 (sec) , antiderivative size = 2856, normalized size of antiderivative = 8.63

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2
*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sq
rt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2
500*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 -
 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8
- 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^
2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)))
- sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 -
4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c
^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*...
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.90.7 Maxima [F]

$$\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2}\, dx = \int \frac{x^{10}}{(cx^5 + bx^3 + ax)^2}\, dx$$

input
```
integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (
b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*
b^2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2
- 4*a*c^3) + x/c^2
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3335 vs. $2(285) = 570$.

Time = 0.99 (sec) , antiderivative size = 3335, normalized size of antiderivative = 10.08

$$\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2}\, dx = \text{Too large to display}$$

input
```
integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^
2*c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c
^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c
)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9
- (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c...
```

3.90.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 7599, normalized size of antiderivative = 22.96

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
int(x^10/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c -
 b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*
c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c
^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*
c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3
840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 -
 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25
*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*
c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (
x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))
/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11...
```

3.90. $\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$

3.91 $\int \frac{x^9}{(ax+bx^3+cx^5)^2} dx$

3.91.1 Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2} dx = -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}} + \frac{\log(a+bx^2+cx^4)}{4c^2}$$

output
```
-1/2*b*x^2/c/(-4*a*c+b^2)+1/2*x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
+1/2*b*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^
2)^(3/2)+1/4*ln(c*x^4+b*x^2+a)/c^2
```

3.91.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2} dx = \frac{\frac{2(-2a^2c+b^3x^2+ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+bx^2+cx^4)}{4c^2}$$

input
```
Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b
^2 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

3.91.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1434, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \; 1434$$

$$\frac{1}{2}\int \frac{x^6}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 1164$$

$$\frac{1}{2}\left(\frac{x^4(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{x^2(bx^2+4a)}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\left(\frac{x^4(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \left(\frac{b}{c}-\frac{(b^2-4ac)x^2+ab}{c(cx^4+bx^2+a)}\right)dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{x^4(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx^2+cx^4)}{2c^2}+\frac{bx^2}{c}}{b^2-4ac}\right)$$

input
```
Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x^2)/c - (b
*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4
*a*c]) - ((b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*c^2))/(b^2 - 4*a*c))/2
```

3.91.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.91.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\frac{b(3ac-b^2)x^2}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{2c\,x^4+2b\,x^2+2a}+\frac{\frac{(4ac-b^2)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$	179
risch	Expression too large to display	1017

input
```
int(x^9/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x
^4+b*x^2+a)+1/2/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^4+b*x^2+a)+2*(-a*b
-1/2*(4*a*c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2)))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(120) = 240$.

Time = 0.30 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.02

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(b^5-7\,ab^3c+12\,a^2bc^2)x^2+((b^3c-6\,abc^2)x^4+ab^3-6\,a^2bc+(b^4-6\,ab^2}{4\,(ab^4c^2-8\,a^2b^2}\right.$$

input
```
integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*
c^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3
 + 16*a^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 -
 7*a*b^3*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^
2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)
*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)
*x^2)]
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.91.7 Maxima [F]

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^9}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*
c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - integrate(-((b^2 - 4*a*c
)*x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} - \frac{b^2cx^4-4\,ac^2x^4-b^3x^2+2\,abcx^2-ab^2}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)} + \frac{\log\left(cx^4+bx^2+a\right)}{4\,c^2}$$

input
```
integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 -
4*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*
a*b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x
^4 + b*x^2 + a)/c^2
```

3.91.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 1336, normalized size of antiderivative = 10.12

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx$$

$$= \frac{\frac{a\,(2\,a\,c-b^2)}{2\,c^2\,(4\,a\,c-b^2)}+\frac{b\,x^2\,(3\,a\,c-b^2)}{2\,c^2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} - \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(-128\,a^3\,c^3+96\,a^2\,b^2\,c^2-24\,a\,b^4\,c+2\,b^6\right)}{2\,\left(256\,a^3\,c^5-192\,a^2\,b^2\,c^4+48\,a\,b^4\,c^3-4\,b^6\,c^2\right)}$$

$$+ \frac{b\,\mathrm{atan}\left(\frac{\left(8\,a\,c^3\,{\left(4\,a\,c-b^2\right)}^3-2\,b^2\,c^2\,{\left(4\,a\,c-b^2\right)}^3\right)\left(x^2\left(\frac{\frac{b\left(\frac{6\,b^3\,c^2-28\,a\,b\,c^3}{4\,a\,c^3-b^2\,c^2}+\frac{\left(8\,b^3\,c^4-32\,a\,b\,c^5\right)\,\left(-128\,a^3\,c^3+96\,a^2\,b^2\,c^2-24\,a\,b^4\,c+2\,b^6\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)\,\left(256\,a^3\,c^5-192\,a^2\,b^2\,c^4+48\,a\,b^4\,c^3-4\,b^6\,c^2\right)}\right)\left(6\,a\,\cdots\right.}{8\,c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}}{a\,\left(4\,a\,c\cdots\right.}\right.\right.}{\cdots}\right)}{\cdots}$$

input
```
int(x^9/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4
*a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a
^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*
b^4*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^
2*(4*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2)
 + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*
b^4*c))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 1
92*a^2*b^2*c^4)))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c
^4 - 32*a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*
a*b^4*c))/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4
*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3
- 5*a*b*c)/(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2
*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 -
24*a*b^4*c))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^
3 - 192*a^2*b^2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((
b^3*c^4)/2 - 2*a*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b
^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*
(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*
c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (...
```

3.91. $\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx$

3.92 $\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 271

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = -\frac{bx}{2c(b^2-4ac)}+\frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2-6ac+\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*b*x/c/(-4*a*c+b^2)+1/2*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1
/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-6*a*c-b*(-8
*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(
b^2-6*a*c+b*(-8*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.04

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{-\frac{2\sqrt{c}x(b^2x^2+a(b-2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4}\right.}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{}}}}{4c^{3/2}}$$

input
```
Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2
*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(4*c^(3/2))
```

3.92.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1440, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{x^6}{(a+bx^2+cx^4)^2}dx$$
$$\Big\downarrow \ 1440$$
$$\frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^2(bx^2+6a)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \; 1602$$

$$\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx}{c}-\frac{\int \frac{\left(b^2-6ac\right)x^2+ab}{cx^4+bx^2+a}dx}{c}}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx}{c}-\frac{\frac{1}{2}\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}}{2\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx}{c}-\frac{\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{2\left(b^2-4ac\right)}$$

input
```
Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x)/c - (((
b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt
[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4*a*c))
```

3.92.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.92.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.56

method	result
risch	$\frac{-\frac{\left(2ac-b^2\right)x^3}{2c\left(4ac-b^2\right)}+\frac{abx}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{\left(6ac-b^2\right)\_R^2}{4ac-b^2}-\frac{ab}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{\left(2ac-b^2\right)x^3}{2c\left(4ac-b^2\right)}+\frac{abx}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+8abc-b^3\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}\cdots\right.}{4c\sqrt{-4a\cdots}}}{4ac-b^2}$

input
```
int(x^8/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+
a)+1/4/c*sum(((6*a*c-b^2)/(4*a*c-b^2)*_R^2-a*b/(4*a*c-b^2))/(2*_R^3*c+_R*b
)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(227) = 454$.

Time = 0.34 (sec) , antiderivative size = 2257, normalized size of antiderivative = 8.33

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
 a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*lo
g((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5
*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^
4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a
^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^
3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^
2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 -
 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c
+ 324*a^3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*
a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 +
768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c...
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.92.7 Maxima [F]

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^8}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^
2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b
)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. $2(227) = 454$.

Time = 0.94 (sec) , antiderivative size = 2736, normalized size of antiderivative = 10.10

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
 - 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*
b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b
^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3...
```

3.92.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 6293, normalized size of antiderivative = 23.22

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
- ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/
(a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*
a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*
a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1
/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3
 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 -
 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*
b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*
a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*
a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*
c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4
*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3...
```

3.92. $\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$

3.93 $\int \frac{x^7}{(ax+bx^3+cx^5)^2} \, dx$

3.93.1 Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2} \, dx = \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*a*arctanh((2*c*x^2+b)/(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2} \, dx = \frac{b^2x^2+a(b-2cx^2)}{2c(-b^2+4ac)(a+bx^2+cx^4)} + \frac{2a\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*
a*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.93.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1434, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \; 1434$$

$$\frac{1}{2}\int \frac{x^4}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 1153$$

$$\frac{1}{2}\left(\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2a\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(\frac{4a\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

3.93.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.93.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x^2}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{2c\,x^4+2b\,x^2+2a}+\frac{2a\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x^2}{2c\left(4ac-b^2\right)}+\frac{ab}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{a\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{a\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)x^2-8c\,a^2+2b\right.}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int(x^7/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a
/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(72) = 144$.

Time = 0.30 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.22

$$\int \frac{x^7}{(ax + bx^3 + cx^5)^2} dx = \left[-\frac{ab^3 - 4a^2bc + (b^4 - 6ab^2c + 8a^2c^2)x^2 + 2(ac^2x^4 + abcx^2 + a^2c)\sqrt{b^2 - 4ac}\log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2}{cx^4 + bx^2 + a}\right.}{2(ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x^2}\right. - \frac{ab^3 - 4a^2bc + (b^4 - 6ab^2c + 8a^2c^2)x^2 - 4(ac^2x^4 + abcx^2 + a^2c)\sqrt{-b^2 + 4ac}\arctan\left(-\frac{(2cx^2 + b)\sqrt{-b^2 + 4}}{b^2 - 4ac}\right.}{2(ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x^2)}$$

input
```
integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^
4 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
- 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c
- 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 +
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4
- 6*a*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2
 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*
c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4
+ (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

3.93.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(70) = 140$.

Time = 0.81 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.62

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = -a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right) + a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right) + \frac{ab+x^2(-2ac+b^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

input
```
integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
-a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (1
6*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*
(-2*a*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2)
 + x**2*(8*a*b*c**2 - 2*b**3*c))
```

3.93.7 Maxima [F]

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^7}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-2*a*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) - 1/2*((b^2 - 2*a*c
)*x^2 + a*b)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a
*b*c^2)*x^2)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = -\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x^2-2\,acx^2+ab}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

input
```
integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4
*a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a
*c^2))
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.40

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx = -\frac{\frac{x^2\,(2\,a\,c-b^2)}{2\,c\,(4\,a\,c-b^2)}-\frac{a\,b}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} - \frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\left(\frac{4\,a\,c^2}{(4\,a\,c-b^2)^{7/2}}+\frac{4\,a\,(b^3\,c^2-4\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{(4\,a\,c-b^2)^{13/2}}\right)(4\,a\,c-b^2)^4}{8\,a^2\,c^2}\right)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int(x^7/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
- ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(
(4*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c)
)/(4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

3.94 $\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx$

3.94.1 Optimal result

Integrand size = 20, antiderivative size = 237

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx=\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.99

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \frac{1}{4}\left(\frac{2(2ax+bx^3)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

input
```
Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2
- 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/4
```

3.94.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 1440, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \; 1440$$

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{2a-bx^2}{cx^4+bx^2+a}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{1}{2}\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}$$

input
```
Int[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((b - (b^2 +
4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((b + (b^2 + 4*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c))
```

3.94.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.94.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.52

method	result
risch	$\frac{-\frac{b\,x^{3}}{2\left(4ac-b^{2}\right)}-\frac{ax}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{b\_R^{2}}{4ac-b^{2}}+\frac{2a}{4ac-b^{2}}\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}\right)}{4}$
default	$\frac{-\frac{b\,x^{3}}{2\left(4ac-b^{2}\right)}-\frac{ax}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(\frac{\left(-b\sqrt{-4ac+b^{2}}-4ac-b^{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,c\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(-b\sqrt{-4ac+b^{2}}+4ac+b^{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,c\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{4ac-b^{2}}$

input
```
int(x^6/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4*sum((-b/(4*a*
c-b^2)*_R^2+2*a/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^
2*b+a))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1668 vs. 2(193) = 386.

Time = 0.28 (sec) , antiderivative size = 1668, normalized size of antiderivative = 7.04

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^
5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a
*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*
b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^
3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 -
 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(
-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sq
rt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4
+ a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c -
12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3...
```

3.94.6 Sympy [A] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.25

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \frac{-2ax-bx^3}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

$$+\text{RootSum}\Big(t^4\cdot(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c^5-327680a^3b^6c^4+61440a^2b^8c^3-6144ab$$

input
```
integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
(-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2
*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**
2*c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c
**3 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*
a**3*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b
**2*c + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_
t**3*a**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a*
*2*c**2 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

3.94.7 Maxima [F]

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^6}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a
*c)
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2132 vs. $2(193) = 386$.

Time = 0.88 (sec) , antiderivative size = 2132, normalized size of antiderivative = 9.00

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/16*(2*b^7*c^2
- 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^
4 - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2
*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^...
```

3.94.9 Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 4973, normalized size of antiderivative = 20.98

$$\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
int(x^6/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
- atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^
4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c -
b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(3
2*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*
c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5
*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 51
2*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^
3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(
b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-
(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^
3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a
^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a
^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b
^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*
a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^
4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b
*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c -
b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)...
```

3.94. $\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx$

3.95 $\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx$

3.95.1 Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx = \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx = \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x
^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.95.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1434, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^3}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1159}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

3.95.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.95.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

method	result
default	$\frac{-b\,x^2-2a}{2(4ac-b^2)(c\,x^4+b\,x^2+a)}-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{b\,x^2}{2\left(4ac-b^2\right)}-\frac{a}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{b\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)}{2(-4ac+b^2)^{\frac{3}{2}}}-\frac{b\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)x^2-8c\,a^2+2b^2a\right.}{2(-4ac+b^2)^{\frac{3}{2}}}$

input
```
int(x^5/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(69) = 138$.

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.80

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \left[\frac{2\,ab^2 - 8\,a^2c + (b^3 - 4\,abc)x^2 - (bcx^4 + b^2x^2 + ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^4 + (b^5 - 8\,ab^3c + 16\,a^2bc^2)x^2)},\right.$$

input
```
integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*
x^2 + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(63) = 126$.

Time = 0.69 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.59

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} + \frac{-2a-bx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

input
```
integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c -
b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c -
b**2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16
*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2
)**3) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**
2)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b
**3))
```

3.95.7 Maxima [F]

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^5}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
b*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) + 1/2*(b*x^2 + 2*a)/((
b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bx^2+2\,a}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

input
```
integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx = \frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}} - \frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{b^2\,c^2}{a\,(4\,a\,c-b^2)^{7/2}} + \frac{b^2\,(2\,b^3\,c^2-8\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{2\,a\,(4\,a\,c-b^2)^{13/2}}\right)}{2\,b^2\,c^2}\right)}{(4\,a\,c-b^2)^{3/2}} - \frac{\frac{a}{4\,a\,c-b^2} + \frac{b\,x^2}{2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}$$

input
```
int(x^5/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
(b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c
^2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)
)/(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a
*c - b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.96 $\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx$

3.96.1 Optimal result

Integrand size = 20, antiderivative size = 221

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*arctan(x*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \frac{-bx-2cx^3}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(
2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.96.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 1439, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \ 1439$$

$$\frac{\int \frac{b-2cx^2}{cx^4+bx^2+a}dx}{2(b^2-4ac)} - \frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

$$\downarrow 1480$$

$$\frac{-c\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-c\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\downarrow 218$$

$$\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
-1/2*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-((Sqrt[2]*S
qrt[c]*(1 - (2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(1 + (
2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*(b^2 - 4*a*c))
```

3.96.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.96.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\frac{c\,x^3}{4ac-b^2}+\frac{bx}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{2c\_R^2}{4ac-b^2}-\frac{b}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{4}$
default	$16c^2\left(\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(-b+\frac{\sqrt{-4ac+b^2}}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}+\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right.}{}}{4c(4ac-b^2)\sqrt{-}}\right.$

input
```
int(x^4/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(c/(4*a*c-b^2)*x^3+1/2*b/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4*sum((2*c/(4*a*
c-b^2)*_R^2-b/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*
b+a))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(180) = 360$.

Time = 0.30 (sec) , antiderivative size = 1680, normalized size of antiderivative = 7.60

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2
*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c +
4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^
2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c +
 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - s
qrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*s
qrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3
)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*
sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4
*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 -
 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*
b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6...
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.96.7 Maxima [F]

$$\int \frac{x^4}{(ax + bx^3 + cx^5)^2} dx = \int \frac{x^4}{(cx^5 + bx^3 + ax)^2} dx$$

input
```
integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a
*b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*
a*c)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1970 vs. 2(180) = 360.

Time = 0.88 (sec) , antiderivative size = 1970, normalized size of antiderivative = 8.91

$$\int \frac{x^4}{(ax + bx^3 + cx^5)^2} dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2
- 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c
^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*
(b^2 - 4*a*c)^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*
(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3...
```

3.96.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 4854, normalized size of antiderivative = 21.96

$$\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^
(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^1
2 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512
*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/
(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^
6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2
*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 +
768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c
^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4
 - 6144*a^6*b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*
c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 -
 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8
*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*
b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2
*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c ...
```

3.96. $\int \frac{x^4}{(ax+bx^3+cx^5)^2} \, dx$

3.97 $\int \frac{x^3}{(ax+bx^3+cx^5)^2} dx$

3.97.1 Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2} dx = -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2} dx = -\frac{\frac{b+2cx^2}{a+bx^2+cx^4} + \frac{4c\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)}$$

input
```
Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
-1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-
b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

3.97.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1432, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{1}{2}\int \frac{1}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{1}{2}\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

3.97.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

3.97.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

method	result	si
default	$\frac{2c\,x^2+b}{2(4ac-b^2)(c\,x^4+b\,x^2+a)}+\frac{2c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	7
risch	$\frac{\frac{c\,x^2}{4ac-b^2}+\frac{b}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{c\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{c\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)x^2-8c\,a^2+2b^2a\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	1

input
```
int(x^3/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 4.88

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx = \left[-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2+2\,(c^2x^4+bcx^2+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}, -\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2-4\,(c^2x^4+bcx^2+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\right]$$

input
```
integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*
c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 +
 b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 +
b*c*x^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(66) = 132$.

Time = 0.68 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.61

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx =$$
$$-c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+\frac{b+2cx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

input
```
integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
-c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (
16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x
**2)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2
*b**3))
```

3.97.7 Maxima [F]

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^3}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-2*c*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) - 1/2*(2*c*x^2 + b)
/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx = -\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx^2+b}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

input
```
integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4
*a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.32

$$\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx = \frac{\frac{b}{2\,(4\,a\,c-b^2)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a} - \frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}} - \frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{4\,c^4}{a\,(4\,a\,c-b^2)^{7/2}}+\frac{4\,c^2\,(b^3\,c^2-4\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{a\,(4\,a\,c-b^2)^{13/2}}\right)}{8\,c^4}\right)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int(x^3/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
(b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*a
tan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a
*(4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(
4*a*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

3.98 $\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx$

3.98.1 Optimal result

Integrand size = 20, antiderivative size = 252

$$\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx = \frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1
/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*ar
ctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b
*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

input
```
Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

3.98.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {9, 1405, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\,(b^2-4ac)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\,(b^2-4ac)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\,(b^2-4ac)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

input
```
Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sq
rt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2
- 4*a*c))
```

3.98.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.98.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{b c x^3}{2a(4ac-b^2)}+\frac{(2ac-b^2)x}{2a(4ac-b^2)}}{c x^4+b x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(-\frac{bc\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_R b}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(-b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(b^2-12ac+b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,(4ac-b^2)}-\frac{\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}\cdots}{\cdots}\right.$

input
```
int(x^2/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a*b*c/(4*a*c-b^2)*x^3+1/2*(2*a*c-b^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+
a)+1/4/a*sum((-b*c/(4*a*c-b^2)*_R^2+(6*a*c-b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*
b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. 2(206) = 412.

Time = 0.36 (sec) , antiderivative size = 2309, normalized size of antiderivative = 9.16

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b
^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*
b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b
^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*
c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2
)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 -
12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^
2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 6
72*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 -
 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/...
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^2}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a
^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c
*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. 2(206) = 412.

Time = 0.99 (sec) , antiderivative size = 2682, normalized size of antiderivative = 10.64

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) +
1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*
b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2
 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 6...
```

3.98.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 6404, normalized size of antiderivative = 25.41

$$\int \frac{x^2}{(ax+bx^3+cx^5)^2} \, dx = \text{Too large to display}$$

input
```
int(x^2/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3
 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a
*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 -
 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^
5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^
6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^
4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 +
 b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*
c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5
)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^...
```

3.98. $\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx$

3.99 $\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx$

3.99.1 Optimal result

Integrand size = 18, antiderivative size = 122

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a+bx^2+cx^4)}{4a^2}$$

output
```
1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*b*(-6*a*c+b^2)*
arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1
/4*ln(c*x^4+b*x^2+a)/a^2
```

3.99.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{\frac{2a(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + 4\log(x) - \frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2\cdots}\right.}{(b^2-4a\cdots}}{4a^2}$$

input
```
Integrate[x/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log
[x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*L
og[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c
 - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

3.99.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {9, 1434, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax+bx^3+cx^5)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{1}{x^2(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{1}{2}\left(\frac{-2ac+b^2+bcx^2}{a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{b^2+cx^2b-4ac}{x^2(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{b^2+cx^2b-4ac}{x^2(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x^2)(b^2-4ac)}{a}-\frac{(b^2-4ac)\log(a+bx^2+cx^4)}{2a}}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[x/(a*x + b*x^3 + c*x^5)^2,x]
```

output
```
((b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*(b^2
- 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) +
 ((b^2 - 4*a*c)*Log[x^2])/a - ((b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*a)
)/(a*(b^2 - 4*a*c)))/2
```

3.99.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.99.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.52

method	result
default	$\frac{\ln(x)}{a^2}-\frac{\frac{\frac{abc\,x^2}{4ac-b^2}-\frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(4a\,c^2-b^2c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(5abc-b^3-\frac{\left(4a\,c^2-b^2c\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{bc\,x^2}{2a\left(4ac-b^2\right)}+\frac{2ac-b^2}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-b^6a^2\right)\_Z^2+\left(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\_Z+\right.}\right.}{}$

input
```
int(x/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2-1/2/a^2*((a*b*c/(4*a*c-b^2)*x^2-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^
4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^4+b*x^2+a)+2*(5*a*b
*c-b^3-1/2*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(112) = 224.

Time = 0.32 (sec) , antiderivative size = 813, normalized size of antiderivative = 6.66

$$\int \frac{x}{\left(ax+bx^3+cx^5\right)^2}\,dx$$
$$= \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(ab^3c-4\,a^2bc^2)x^2+((b^3c-6\,abc^2)x^4+ab^3-6\,a^2bc+(b^4-6\,ab^2c)x^2)\sqrt{\cdots}}{}\right.$$

input
```
integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2
+ ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2
)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2
 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a
*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3
*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2
 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4
 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*
a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 +
 (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2)*x^2)]
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(ax + bx^3 + cx^5)^2} \, dx = \text{Timed out}$$

input
```
integrate(x/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.99.7 Maxima [F]

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
 + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-((b^2*c - 4*a*c^2)*x^3 + (b^3 - 5
*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + log(x)/a^2
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.36

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} + \frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\,(cx^4+bx^2+a)(a^2b^2-4\,a^3c)} - \frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2} + \frac{\log\left(x^2\right)}{2\,a^2}$$

input
```
integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 -
4*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*
a*b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) -
 1/4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

3.99.9 Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 5048, normalized size of antiderivative = 41.38

$$\int \frac{x}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
int(x/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c -
b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3
+ 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c
 + 192*a^4*b^2*c^2)) + (b*atan((x^2*((((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^
3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*
(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 26
88*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2
))/(4*a^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 9
6*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7
*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a
^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*
c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^
2*c^2)) + (b*((6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^
3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*
(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -...
```

3.100 $\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$

3.100.1 Optimal result

Integrand size = 16, antiderivative size = 308

$$\begin{aligned}\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx &= -\frac{3b^2-10ac}{2a^2(b^2-4ac)x}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)}\\ &\quad-\frac{\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad+\frac{\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^
2)/x/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^
2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.98

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$$
$$= \frac{-\frac{4}{x}-\frac{2x(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3b^3-16abc-\right.}{}}{4a^2}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)^(-2),x]
```

output
```
(-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 -
 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2
 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

3.100.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1949, 1441, 25, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$$
$$\Big\downarrow \; 1949$$
$$\int \frac{1}{x^2(a+bx^2+cx^4)^2}dx$$
$$\Big\downarrow \; 1441$$
$$\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3b^2+3cx^2b-10ac}{x^2(cx^4+bx^2+a)}dx}{2a(b^2-4ac)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2(cx^4+bx^2+a)}dx}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{1604}$$

$$\frac{-\frac{\int \frac{c(3b^2-10ac)x^2+b(3b^2-13ac)}{cx^4+bx^2+a}dx}{a}-\frac{3b^2-10ac}{ax}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{-\frac{\frac{1}{2}c\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{c\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}}{a}-\frac{3b^2-10ac}{ax}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{ax}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(a*x + b*x^3 + c*x^5)^(-2),x]
```

output
```
(b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (-((3*
b^2 - 10*a*c)/(a*x)) - ((Sqrt[c]*(3*b^2 - 10*a*c + (3*b^3)/Sqrt[b^2 - 4*a*
c] - (16*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^3
- 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b
^2 - 4*a*c]]))/a)/(2*a*(b^2 - 4*a*c))
```

3.100. $\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

3.100.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

method	result
default	$-\frac{1}{a^{2} x}-\frac{\frac{\frac{c\left(2 a c-b^{2}\right) x^{3}}{8 a c-2 b^{2}}+\frac{b\left(3 a c-b^{2}\right) x}{8 a c-2 b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{2 c\left(\frac{\left(10 a c\sqrt{-4 a c+b^{2}}-3 b^{2}\sqrt{-4 a c+b^{2}}-16 a b c+3 b^{3}\right)\sqrt{2}\,\arctan\left(\frac{c x\sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{8\sqrt{-4 a c+b^{2}}\,\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}-\frac{\left(10 a c\sqrt{-4 a c\cdots}\right.}{\cdots}\right.}{4 a c-b^{2}}}{a^{2}}$
risch	$\frac{-\frac{c\left(10 a c-3 b^{2}\right) x^{4}}{2 a^{2}\left(4 a c-b^{2}\right)}-\frac{b\left(11 a c-3 b^{2}\right) x^{2}}{2\left(4 a c-b^{2}\right) a^{2}}-\frac{1}{a}}{x\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4096 a^{11} c^{6}-6144 a^{10} b^{2} c^{5}+3840 a^{9} b^{4} c^{4}-1280 a^{8} b^{6} c^{3}+240 a^{7} b^{8} c^{2}-24 a^{6} b^{10} c+a^{5} b^{12}\cdots\right.\right.}\right.}{\cdots}$

input
```
int(1/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-1/a^2*((1/2*c*(2*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*b*(3*a*c-b^2)/(4*a*
c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-
3*b^2*(-4*a*c+b^2)^(1/2)-16*a*b*c+3*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))-1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4*a*c+b^2)^(1/2)+16*a*b*c-3*b
^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2912 vs. 2(260) = 520.

Time = 0.45 (sec) , antiderivative size = 2912, normalized size of antiderivative = 9.45

$$\int \frac{1}{\left(ax+bx^3+cx^5\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*
c)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^
3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 -
420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqr
t((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^
4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 -
 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b
^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486
*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200
*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c
^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a
^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3
)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*
b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*
c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sq
rt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12
*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + ...
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.100.7 Maxima [F]

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{1}{(cx^5+bx^3+ax)^2}\,dx$$

input
```
integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^
2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4
*a^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/
(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3087 vs. 2(260) = 520.

Time = 0.69 (sec) , antiderivative size = 3087, normalized size of antiderivative = 10.02

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*
a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 -
 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^
4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 +
 (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*...
```

3.100.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 7555, normalized size of antiderivative = 24.53

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a*x + b*x^3 + c*x^5)^2,x)
```

output
```
- atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 20
77*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7
*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2
)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b
^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7
+ x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25
*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*
c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(10
48576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9
*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7))
+ x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^
8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(
-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9
*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^...
```

3.100. $\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$

3.101 $\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx$

3.101.1 Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx = -\frac{b^2-3ac}{a^2(b^2-4ac)x^2}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \frac{(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3}+\frac{b\log(a+bx^2+cx^4)}{2a^3}$$

output
```
(3*a*c-b^2)/a^2/(-4*a*c+b^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^
2/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^
2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+1/2*b*ln(c*x^4+b*x^2+a)/a^3
```

3.101.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.53

$$\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx = \frac{-\frac{a}{x^2}-\frac{a(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}-4b\log(x)+\frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{(-b^4+}{}}{2a^3}$$

input
```
Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

output
```
(-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqr
t[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*
c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^
2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(3/2))/(2*a^3)
```

3.101.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {9, 1434, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{1}{2}\left(\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2\left(b^2+cx^2b-3ac\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{2\int \frac{b^2+cx^2b-3ac}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\left(\frac{2\int \left(\frac{b^2-3ac}{ax^4}+\frac{b^4-5acb^2+c(b^2-4ac)x^2b+3a^2c^2}{a^2(cx^4+bx^2+a)}+\frac{4abc-b^3}{a^2x^2}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\left(-\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x^2)(b^2-4ac)}{a^2}+\frac{b(b^2-4ac)\log(a+bx^2+cx^4)}{2a^2}-\frac{b^2-3ac}{ax^2}\right)}{a\left(b^2-4ac\right)}+\frac{-2ac+}{ax^2\left(b^2-4ac\right.}\right.$$

input
```
Int[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) + (2*(-
((b^2 - 3*a*c)/(a*x^2)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[x^
2])/a^2 + (b*(b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*a^2)))/(a*(b^2 - 4*a
*c)))/2
```

3.101.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.101.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

method	result
default	$-\frac{1}{2a^2x^2}-\frac{2b\ln(x)}{a^3}-\frac{\frac{\frac{ac\left(2ac-b^2\right)x^2}{4ac-b^2}+\frac{ab\left(3ac-b^2\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-4ab\,c^2+b^3c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{c}+\frac{4\left(3a^2c^2-5a\,b^2c+b^4-\frac{\left(-4ab\,c^2+b^3c\right)b}{2c}\right)\text{arctan}}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^3}$
risch	$\frac{-\frac{c\left(3ac-b^2\right)x^4}{a^2\left(4ac-b^2\right)}-\frac{b\left(7ac-2b^2\right)x^2}{2\left(4ac-b^2\right)a^2}-\frac{1}{2a}}{x^2(c\,x^4+b\,x^2+a)}-\frac{2b\ln(x)}{a^3}+\left(\sum_{\_R=\text{RootOf}\left(\left(64a^6c^3-48b^2a^5c^2+12a^4b^4c-b^6a^3\right)\_Z^2+(-64b\,c^3a^3+48b^3c^2a\right)}\right.$

input
```
int(1/x/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2-2*b*ln(x)/a^3-1/2/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+a*b*(
3*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3
*c)/c*ln(c*x^4+b*x^2+a)+2*(3*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(154) = 308$.

Time = 0.37 (sec) , antiderivative size = 1007, normalized size of antiderivative = 6.22

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx$$
$$= \left[-\frac{a^2b^4-8\,a^3b^2c+16\,a^4c^2+2\,(ab^4c-7\,a^2b^2c^2+12\,a^3c^3)x^4+(2\,ab^5-15\,a^2b^3c+28\,a^3bc^2)x^2+((b^4c-\cdots}{\cdots}\right.$$
$$-\frac{a^2b^4-8\,a^3b^2c+16\,a^4c^2+2\,(ab^4c-7\,a^2b^2c^2+12\,a^3c^3)x^4+(2\,ab^5-15\,a^2b^3c+28\,a^3bc^2)x^2+2\,((b^4c-\cdots}{\cdots}$$

input
```
integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 1
2*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6
*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4
 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*
x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
 - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^
2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 +
 a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3
*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8
*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (
2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a
^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c +
 6*a^3*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*l
og(c*x^4 + b*x^2 + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6
 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*
x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 -...
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.101.7 Maxima [F]

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{\left(cx^5+bx^3+ax\right)^2 x}\,dx$$

input
```
integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b^2*c - 3*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*x^2)/(
(a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4
*c)*x^2) - 2*integrate(-((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 5*a*b^2*c + 3*a^
2*c^2)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c) - 2*b*log(x)/a^3
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx &= \frac{(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}} \\ &\quad - \frac{2\,b^2cx^4-6\,ac^2x^4+2\,b^3x^2-7\,abcx^2+ab^2-4\,a^2c}{2\,(cx^6+bx^4+ax^2)(a^2b^2-4\,a^3c)} \\ &\quad + \frac{b\log\left(cx^4+bx^2+a\right)}{2\,a^3} - \frac{b\log\left(x^2\right)}{a^3}\end{aligned}$$

input
```
integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a
^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2
*b^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^
2 - 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

3.101.9 Mupad [B] (verification not implemented)

Time = 11.26 (sec) , antiderivative size = 5491, normalized size of antiderivative = 33.90

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a*x + b*x^3 + c*x^5)^2),x)
```

output
```
(log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) -
(x^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2
*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^
9*b^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4
*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a
^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a
^2*b*c^6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*
b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*
b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*
c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c
^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2
*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*
c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c
^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c
^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2
- 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*...
```

3.102 $\int \frac{1}{x^2(ax+bx^3+cx^5)^2}\,dx$

3.102.1 Optimal result

Integrand size = 20, antiderivative size = 361

$$\begin{aligned}
&\int \frac{1}{x^2(ax+bx^3+cx^5)^2}\,dx \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)x^3}+\frac{b(5b^2-19ac)}{2a^3(b^2-4ac)x}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)} \\
&\quad+\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad-\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2-b(5b^2-19ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

output
```
1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^3+1/2*b*(-19*a*c+5*b^2)/a^3/(-4*a*c+
b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)+1/4*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2
*c+28*a^2*c^2+b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)
*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b
^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 \left(ax+bx^3+cx^5\right)^2} \, dx = \frac{-\frac{4a}{x^3}+\frac{24b}{x}+\frac{6x\left(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{12a^3}$$

input
```
Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

output
```
((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3
*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5
*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4
 + 29*a*b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

3.102.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {9, 1441, 25, 1604, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax+bx^3+cx^5\right)^2} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{1}{x^4 \left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 1441$$

$$\frac{-2ac+b^2+bcx^2}{2ax^3 \left(b^2-4ac\right) \left(a+bx^2+cx^4\right)}-\frac{\int -\frac{5b^2+5cx^2b-14ac}{x^4(cx^4+bx^2+a)}dx}{2a \left(b^2-4ac\right)}$$

$\downarrow$ 25

$$\frac{\int \frac{5b^2+5cx^2b-14ac}{x^4(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 1604

$$\frac{-\frac{\int \frac{3\left(c\left(5b^2-14ac\right)x^2+b\left(5b^2-19ac\right)\right)}{x^2\left(cx^4+bx^2+a\right)}dx}{3a}-\frac{5b^2-14ac}{3ax^3}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{c\left(5b^2-14ac\right)x^2+b\left(5b^2-19ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx}{a}-\frac{5b^2-14ac}{3ax^3}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 1604

$$\frac{-\frac{-\frac{\int \frac{5b^4-24acb^2+c\left(5b^2-19ac\right)x^2b+14a^2c^2}{cx^4+bx^2+a}dx}{a}-\frac{b\left(5b^2-19ac\right)}{ax}}{a}-\frac{5b^2-14ac}{3ax^3}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 1480

$$-\frac{-\frac{\frac{c\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}}{a}}{a}$$

$$2a\left(b^2-4ac\right)$$

$$\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 218

$$-\frac{-\frac{\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{a}$$

$$2a\left(b^2-4ac\right)$$

$$\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

3.102. $\int \frac{1}{x^2(ax+bx^3+cx^5)^2}\,dx$

output
```
(b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4)) + (-1/
3*(5*b^2 - 14*a*c)/(a*x^3) - (-((b*(5*b^2 - 19*a*c))/(a*x)) - ((Sqrt[c]*(5
*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 -
4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^
2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/a)/a)/(2*a*(b^2 - 4*a*c))
```

3.102.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.102.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}-\frac{\frac{-\frac{bc(3ac-b^2)x^3}{2(4ac-b^2)}+\frac{(2a^2c^2-4a\,b^2c+b^4)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-19\sqrt{-4ac+b^2}\,abc+5\sqrt{-4ac+b^2}\,b^3-28a^2c^2+29a\,b^2c-5b^4\right)\sqrt{2}\,\arctan(\ldots}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\ldots\right)}{\ldots}}{\ldots}$
risch	$\frac{\frac{cb(19ac-5b^2)x^6}{2(4ac-b^2)a^3}-\frac{(14a^2c^2-62a\,b^2c+15b^4)x^4}{6a^3(4ac-b^2)}+\frac{5b\,x^2}{3a^2}-\frac{1}{3a}}{x^3(c\,x^4+b\,x^2+a)}+\frac{\left(\_R=\text{RootOf}\left(\left(4096a^{13}c^6-6144a^{12}b^2c^5+3840a^{11}b^4c^4-1280a^{10}b^6c^3+240a^9b\ldots\right.\right.\right.}{\ldots}$

input
```
int(1/x^2/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3+2/a^3*b/x-1/a^3*((-1/2*b*c*(3*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*(2
*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/
8*(-19*(-4*a*c+b^2)^(1/2)*a*b*c+5*(-4*a*c+b^2)^(1/2)*b^3-28*a^2*c^2+29*a*b
^2*c-5*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(5*b^4-29*a*b^2*c+2
8*a^2*c^2+5*(-4*a*c+b^2)^(1/2)*b^3-19*(-4*a*c+b^2)^(1/2)*a*b*c)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3435 vs. $2(311) = 622$.

Time = 0.70 (sec) , antiderivative size = 3435, normalized size of antiderivative = 9.52

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*
x^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2 + 3*sqrt(1/2)*((a^
3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)
*x^3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 +
1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*s
qrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76
686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^
4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^
2*c^2 - 64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*
c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425
*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 -
172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*
a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328
*a^12*b*c^5)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a
*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10
*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5
*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - ...
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.102.7 Maxima [F]

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} \, dx = \int \frac{1}{(cx^5 + bx^3 + ax)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(5*b^3*c - 19*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x^4
 - 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c
^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integ
rate(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + (5*b^3*c - 19*a*b*c^2)*x^2)/(c*x^
4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3651 vs. $2(311) = 622$.

Time = 0.98 (sec) , antiderivative size = 3651, normalized size of antiderivative = 10.11

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c*x^3 - 3*a*b*c^2*x^3 + b^4*x - 4*a*b^2*c*x + 2*a^2*c^2*x)/((a^3*
b^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*(10*a^6*b^9*c^2 - 138*a^7*b^7*c
^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2
- 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^
8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152
*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3...
```

3.102.9 Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 8739, normalized size of antiderivative = 24.21

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a*x + b*x^3 + c*x^5)^2),x)
```

output
```
atan((((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 63
66*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*
c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^1
3*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c
^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(3
20*a^12*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*
c^4 - 468480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 +
2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80
640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4
 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)
^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b
^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10
*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b
^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c
^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572
864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^
12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7
+ 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a
*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^...
```

3.102. $\int \frac{1}{x^2(ax+bx^3+cx^5)^2}\,dx$

3.103 $\int \frac{1}{x^3(ax+bx^3+cx^5)^2}\,dx$

3.103.1 Optimal result

Integrand size = 20, antiderivative size = 219

$$\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx = -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}+\frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{3/2}}+\frac{(3b^2-2ac)\log(x)}{a^4}-\frac{(3b^2-2ac)\log\left(a+bx^2+cx^4\right)}{4a^4}$$

output
```
1/4*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^4+1/2*b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b
^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^4/(c*x^4+b*x^2+a)+1/2*b*(
30*a^2*c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(
-4*a*c+b^2)^(3/2)+(-2*a*c+3*b^2)*ln(x)/a^4-1/4*(-2*a*c+3*b^2)*ln(c*x^4+b*x
^2+a)/a^4
```

3.103.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^3 \left(ax+bx^3+cx^5\right)^2} \, dx$$
$$= \frac{-\frac{a^2}{x^4}+\frac{4ab}{x^2}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}+4(3b^2-2ac)\log(x)-\frac{\left(3b^5-20ab^3c+30a^2bc^2+3b^4\sqrt{b^2-4ac}-14ab^2c\sqrt{b^2-}\right.}{(b^2-4ac)}}{}$$

input
```
Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

output
```
(-(a^2/x^4) + (4*a*b)/x^2 + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2
- 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(3*b^2 - 2*a*c)*
Log[x] - ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 + 3*b^4*Sqrt[b^2 - 4*a*c] - 1
4*a*b^2*c*Sqrt[b^2 - 4*a*c] + 8*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((3*b^5 - 20*a*b^3*c + 30*a^2
*b*c^2 - 3*b^4*Sqrt[b^2 - 4*a*c] + 14*a*b^2*c*Sqrt[b^2 - 4*a*c] - 8*a^2*c^
2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^4)
```

3.103.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {9, 1434, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(ax+bx^3+cx^5\right)^2} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^5 \left(a+bx^2+cx^4\right)^2}dx$$
$$\Big\downarrow \; 1434$$
$$\frac{1}{2}\int \frac{1}{x^6 \left(cx^4+bx^2+a\right)^2}dx^2$$
$$\Big\downarrow \; 1165$$

$$\frac{1}{2}\left(\frac{-2ac+b^2+bcx^2}{ax^4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3b^2+3cx^2b-8ac}{x^6(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{3b^2+3cx^2b-8ac}{x^6(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{ax^4(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{3b^2-8ac}{ax^6}+\frac{-c(3b^4-14acb^2+8a^2c^2)x^2-b(3b^4-17acb^2+19a^2c^2)}{a^3(cx^4+bx^2+a)}+\frac{(b^2-4ac)(3b^2-2ac)}{a^3x^2}+\frac{b(11ac-3b^2)}{a^2x^4}\right)dx^2}{a(b^2-4ac)}+\frac{-2ac+}{ax^4(b^2-4ac}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\frac{\log(x^2)(b^2-4ac)(3b^2-2ac)}{a^3}-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx^2+cx^4)}{2a^3}+\frac{b(3b^2-11ac)}{a^2x^2}+\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-}{a(b^2-4ac)}\right.$$

input
```
Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*(a + b*x^2 + c*x^4)) + (-1/2
*(3*b^2 - 8*a*c)/(a*x^4) + (b*(3*b^2 - 11*a*c))/(a^2*x^2) + (b*(3*b^4 - 20
*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt
[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[x^2])/a^3 - ((b^2 - 4*
a*c)*(3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(2*a^3))/(a*(b^2 - 4*a*c)))/2
```

3.103.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.103.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.20

method	result
default	$-\frac{1}{4a^2x^4}+\frac{(-2ac+3b^2)\ln(x)}{a^4}+\frac{b}{a^3x^2}+\frac{\frac{\frac{acb\left(3ac-b^2\right)x^2}{4ac-b^2}-\frac{a\left(2a^2c^2-4a\,b^2c+b^4\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(8a^2c^3-14b^2a\,c^2+3b^4c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(19a\right.}{}}{2a^4}$
risch	$\frac{\frac{bc\left(11ac-3b^2\right)x^6}{2a^3\left(4ac-b^2\right)}-\frac{\left(8a^2c^2-25a\,b^2c+6b^4\right)x^4}{4a^3\left(4ac-b^2\right)}+\frac{3b\,x^2}{4a^2}-\frac{1}{4a}}{x^4(c\,x^4+b\,x^2+a)}-\frac{2\ln(x)c}{a^3}+\frac{3b^2\ln(x)}{a^4}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(64a^7c^3-48a^6b^2c^2+12a^5b^4c-b^6a^4\right)\right.}\right.}{}$

input
```
int(1/x^3/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/x^4+(-2*a*c+3*b^2)*ln(x)/a^4+1/a^3*b/x^2+1/2/a^4*((a*c*b*(3*a*c-b
^2)/(4*a*c-b^2)*x^2-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^4+b*x^2+
a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*ln(c*x^4+b*x^2+a)
+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(205) = 410$.

Time $= 0.45$ (sec) , antiderivative size $= 1242$, normalized size of antiderivative $= 5.67$

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

output
```
[-1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2
+ 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c
^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 + ((3*b^5*c - 20*a*
b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 +
(3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
b*x^2 + a)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8
+ (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26
*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*
a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 6
4*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*
a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a
^6*b^2*c + 16*a^7*c^2)*x^4), -1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*
(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c
+ 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*
c^2)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*
b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)
*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)
) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^...
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)
```

output
```
Timed out
```

3.103.7 Maxima [F]

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = \int \frac{1}{(cx^5 + bx^3 + ax)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

output
```
1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c^2)*x^4 -
 a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x
^8 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4) - integrate(((3*
b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^3 + (3*b^5 - 17*a*b^3*c + 19*a^2*b*c^2
)*x)/(c*x^4 + b*x^2 + a), x)/(a^4*b^2 - 4*a^5*c) + (3*b^2 - 2*a*c)*log(x)/
a^4
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = -\frac{(3\,b^5 - 20\,ab^3c + 30\,a^2bc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^4b^2 - 4\,a^5c)\sqrt{-b^2+4\,ac}} + \frac{3\,b^4cx^4 - 14\,ab^2c^2x^4 + 8\,a^2c^3x^4 + 3\,b^5x^2 - 12\,ab^3cx^2 + 2\,a^2bc^2x^2 + 5\,ab^4 - 22\,a^2b^2c + 12\,a^3c^2}{4\,(a^4b^2 - 4\,a^5c)(cx^4 + bx^2 + a)} - \frac{(3\,b^2 - 2\,ac)\log(cx^4 + bx^2 + a)}{4\,a^4} + \frac{(3\,b^2 - 2\,ac)\log(x^2)}{2\,a^4} - \frac{9\,b^2x^4 - 6\,acx^4 - 4\,abx^2 + a^2}{4\,a^4x^4}$$

input
```
integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*x^4 - 14*a
*b^2*c^2*x^4 + 8*a^2*c^3*x^4 + 3*b^5*x^2 - 12*a*b^3*c*x^2 + 2*a^2*b*c^2*x^
2 + 5*a*b^4 - 22*a^2*b^2*c + 12*a^3*c^2)/((a^4*b^2 - 4*a^5*c)*(c*x^4 + b*x
^2 + a)) - 1/4*(3*b^2 - 2*a*c)*log(c*x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2 - 2
*a*c)*log(x^2)/a^4 - 1/4*(9*b^2*x^4 - 6*a*c*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^
4)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 11.78 (sec) , antiderivative size = 5999, normalized size of antiderivative = 27.39

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a*x + b*x^3 + c*x^5)^2),x)
```

output
```
(b*atan((x^2*((((((b*((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 +
108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c
 + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^
3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*
b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^
10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*
a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/
2)) - (b*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)*(2560*a^13*b*c^6 + 12*a^9*b^9*c
^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 25
6*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c
 - b^2)^(3/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4
*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^
4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 2
56*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) + (b*((1760*a^7*b*c^8 + 54*a
^3*b^9*c^4 - 657*a^4*b^7*c^5 + 2775*a^5*b^5*c^6 - 4484*a^6*b^3*c^7)/(a^9*b
^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((2240*a^10*b*c^7 -
 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9
*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6
+ 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^
5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*...
```

3.104 $\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx$

3.104.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2x^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

output
```
2/3*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 11.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{x\,(a+bx^2+cx^4)}}$$

input
```
Integrate[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(2*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4,
 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])])/(3*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.104.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1977, 1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}dx$$

$$\Big\downarrow \text{1977}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{x}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{\sqrt{x}}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x
^5])
```

3.104.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

rule 1977
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n
 - q) + c*x^(2*(n - q)))^p)   Int[x^(m + p*q)*(a + b*x^(n - q) + c*x^(2*(n
- q)))^p, x], x] /; FreeQ[{a, b, c, m, n, p, q}, x] && EqQ[r, 2*n - q] &&
!IntegerQ[p] && PosQ[n - q]
```

3.104.4 Maple [F]

$$\int \frac{x}{\sqrt{c\,x^{5}+b\,x^{3}+a x}}dx$$

input
```
int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

output
```
int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

3.104.5 Fricas [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^5 + b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)
```

3.104.6 Sympy [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{x}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

input
```
integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.104.7 Maxima [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.104.8 Giac [F]

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{x}{\sqrt{c\,x^5+b\,x^3+a\,x}} \,dx$$

input
```
int(x/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

output
```
int(x/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.105 $\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$

3.105.1 Optimal result

Integrand size = 24, antiderivative size = 380

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = -\frac{2(b^2-3ac)\,x^{3/2}(a+bx^2+cx^4)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt{x}(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}$$
$$+\frac{2\sqrt[4]{a}(b^2-3ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{a}\left(2b^2+\sqrt{a}b\sqrt{c}-6ac\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

output
```
-2/15*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(
c*x^5+b*x^3+a*x)^(1/2)+1/15*(3*c*x^2+b)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+
2/15*a^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/30*a^(
1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^(1/2))*x^(1/2)*((c
*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/
2)
```

3.105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.28

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx=\frac{\sqrt{x}\left(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b+3cx^2)\left(a+bx^2+cx^4\right)-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right.\right.}{}$$

input
```
Integrate[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 +
 c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
+ I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])]))/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a +
b*x^2 + c*x^4)])
```

3.105.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1966, 25, 2000, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \text{1966}$$

$$\frac{\int -\frac{\sqrt{x}\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^5+bx^3+ax}}dx}{15c}+\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\int \frac{\sqrt{x}\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^5+bx^3+ax}}dx}{15c}$$

$\big\downarrow$ 2000

$$\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{2\left(b^2-3ac\right)x^2+ab}{\sqrt{cx^4+bx^2+a}}dx}{15c\sqrt{ax+bx^3+cx^5}}$$

$\big\downarrow$ 1511

$$\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{15c\sqrt{ax+bx^3+cx^5}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{15c\sqrt{ax+bx^3+cx^5}}$$

$\big\downarrow$ 1416

$$\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\int \ldots}{\sqrt{c}}\ldots\right.}{15c\sqrt{ax+bx^3+cx^5}}$$

$\big\downarrow$ 1509

$$\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\left(\ldots\right.}{\ldots}\right.}{15c\sqrt{ax+bx^3+cx^5}}$$

3.105. $\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$

input
```
Int[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(Sqrt[x]*(b + 3*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(15*c) - (Sqrt[x]*Sqrt[a
 + b*x^2 + c*x^4]*((-2*(b^2 - 3*a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[
a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] +
 (a^(1/4)*(Sqrt[a]*b + (2*(b^2 - 3*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2
 + c*x^4])))/(15*c*Sqrt[a*x + b*x^3 + c*x^5])
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 2000
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2
)*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fr
eeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && Pos
Q[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

3.105.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.20

method	result
risch	$\frac{x^{\frac{3}{2}}\left(3cx^2+b\right)\left(cx^4+bx^2+a\right)}{15c\sqrt{x\left(cx^4+bx^2+a\right)}} - \frac{\left(\frac{ab\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4a\cdots}\right)}{ac}}}{2}\cdots\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x^(3/2)*(3*c*x^2+b)*(c*x^4+b*x^2+a)/c/(x*(c*x^4+b*x^2+a))^(1/2)-1/15/
c*(1/4*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(6*a*c-2*b^2)*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))*
(c*x^4+b*x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.97

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((b^2c-3\,ac^2)x^2\sqrt{\frac{b^2-4\,ac}{c^2}}-(b^3-3\,abc)x^2\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+}{2\,ac}}{}$$

input
```
integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(2*sqrt(1/2)*((b^2*c - 3*a*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (b^3 -
 3*a*b*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(
arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt
((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b^2*c - (6*a + b
)*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (2*b^3 - (6*a*b - b^2)*c)*x^2)*sqrt(c
)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt
((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) +
 b^2 - 2*a*c)/(a*c)) - 2*(3*c^3*x^4 + b*c^2*x^2 - 2*b^2*c + 6*a*c^2)*sqrt(
c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x^2)
```

3.105.6 Sympy [F]

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = \int x^{\frac{3}{2}}\sqrt{x\left(a+bx^2+cx^4\right)}\,dx$$

input
```
integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.105.7 Maxima [F]

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = \int \sqrt{cx^5+bx^3+ax}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

3.105.8 Giac [F]

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = \int \sqrt{cx^5+bx^3+ax}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx=\int x^{3/2}\,\sqrt{c\,x^5+b\,x^3+a\,x}\,dx$$

input
```
int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
```

output
```
int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.106 $\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$

3.106.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{(b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

output
```
-1/16*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*
x^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/8*(2*c*x^2
+b)*(c*x^5+b*x^3+a*x)^(1/2)/c/x^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \frac{\sqrt{x(a+bx^2+cx^4)}\left(\sqrt{c}(b+2cx^2)+\frac{(b^2-4ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}-\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(Sqrt[x*(a + b*x^2 + c*x^4)]*(Sqrt[c]*(b + 2*c*x^2) + ((b^2 - 4*a*c)*ArcTa
nh[(Sqrt[c]*x^2)/(Sqrt[a] - Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a + b*x^2 + c*
x^4]))/(8*c^(3/2)*Sqrt[x])
```

3.106.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1965, 1961, 1432, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$$

$$\Big\downarrow \text{1965}$$

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{(b^2-4ac)\int \frac{x^{3/2}}{\sqrt{cx^5+bx^3+ax}}dx}{8c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}(b^2-4ac)\sqrt{a+bx^2+cx^4}\int \frac{x}{\sqrt{cx^4+bx^2+a}}dx}{8c\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}(b^2-4ac)\sqrt{a+bx^2+cx^4}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{16c\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}(b^2-4ac)\sqrt{a+bx^2+cx^4}\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{8c\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}(b^2-4ac)\sqrt{a+bx^2+cx^4}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a*c)*S
qrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

3.106.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

3.106.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

method	result
risch	$\frac{(2c\,x^2+b)\,(c\,x^4+b\,x^2+a)\sqrt{x}}{8c\sqrt{x(c\,x^4+b\,x^2+a)}}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{x}}{16c^{\frac{3}{2}}\sqrt{x(c\,x^4+b\,x^2+a)}}$
default	$\frac{\sqrt{x(c\,x^4+b\,x^2+a)}\left(4c^{\frac{3}{2}}x^2\sqrt{c\,x^4+b\,x^2+a}+4\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)ac-\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)b^2+2b\sqrt{c\,x^4+b\,x^2+}\right.}{16c^{\frac{3}{2}}\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}$

input
```
int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)/c*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)+1/16*(
4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*(c*x^4+
b*x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.80

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^5+bx^3+ax}(2\,c^2x^2+b}{32\,c^2x}\right.$$

input
```
integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5
 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) - 4*sq
rt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x), 1/16*((b^2 - 4
*a*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-
c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c
^2*x^2 + b*c)*sqrt(x))/(c^2*x)]
```

3.106.6 Sympy [F]

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx=\int \sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}\,dx$$

input
```
integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.106.7 Maxima [F]

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx=\int \sqrt{cx^5+bx^3+ax}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x), x)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx=&\ \frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2\,x^2+\frac{b}{c}\right)\\&+\frac{\left(b^2-4\,ac\right)\log\left(\left|2\,\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{3}{2}}}\\&-\frac{b^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-4\,ac\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+2\,\sqrt{a}b\sqrt{c}}{16\,c^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2) - 1/16*(b^2*l
og(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2
*sqrt(a)*b*sqrt(c))/c^(3/2)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx = \int \sqrt{x}\,\sqrt{c\,x^5+b\,x^3+a\,x}\,dx$$

input
```
int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
```

output
```
int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.107 $\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$

3.107.1 Optimal result

Integrand size = 24, antiderivative size = 347

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx = \frac{bx^{3/2}(a+bx^2+cx^4)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$
$$-\frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}$$

output
```
1/3*b*x^(3/2)*(c*x^4+b*x^2+a)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a
*x)^(1/2)+1/3*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)-1/3*a^(1/4)*b*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x
^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4
)/(c*x^5+b*x^3+a*x)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a
^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/
2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4
)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.107.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$
$$= \frac{\sqrt{x}\left(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\right.\right.}{}$$

input
```
Integrate[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

output
```
(Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*
(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt
[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.107.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1968, 2000, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$
$$\Big\downarrow \text{1968}$$
$$\frac{1}{3}\int \frac{\sqrt{x}\left(bx^2+2a\right)}{\sqrt{cx^5+bx^3+ax}}dx+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$

$$\Big\downarrow \text{2000}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{3\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a}{(}}}{}\right.}{}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}$$

input
```
Int[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

3.107. $\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$

output
```
(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 + (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(
-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^
(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c
])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(3*Sqrt[a*x + b*x^3 + c*x^5])
```

3.107.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1968
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Simp[(n - q)*(p/(m + p*(2*n - q) + 1))   Int[x^(m + q)*
(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; Free
Q[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
 + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

rule 2000
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2
)*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fr
eeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && Pos
Q[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

3.107.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.24

method	result
risch	$\frac{x^{\frac{3}{2}}\left(c\,x^{4}+b\,x^{2}+a\right)}{3\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}+\frac{\left(\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{6\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\right.}{}$
default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\left(\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{-4ac+b^{2}}\,c\,x^{5}+\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,bc\,x^{5}+\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{-4ac+b^{2}}\,b\,x^{3}+\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\right.}{}$

input
```
int((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3/2)*(c*x^4+b*x^2+a)/(x*(c*x^4+b*x^2+a))^(1/2)+(1/6*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-1/6*b*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))*(c*x^4+b*x^2+a)^(1/2)*x^(1/2)/(x*(c*
x^4+b*x^2+a))^(1/2)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left(bcx^2\sqrt{\frac{b^2-4\,ac}{c^2}}-b^2x^2\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right) \mid \frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac}) - \sqrt{\frac{1}{2}}\left((bc-\right.}{}$$

input
```
integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(1/2)*(b*c*x^2*sqrt((b^2 - 4*a*c)/c^2) - b^2*x^2)*sqrt(c)*sqrt((c
*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*
a*c)/(a*c)) - sqrt(1/2)*((b*c - 2*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (b^2
+ 2*b*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(a
rcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt(
(b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(c^
2*x^2 + b*c)*sqrt(x))/(c^2*x^2)
```

3.107.6 Sympy [F]

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx = \int \frac{\sqrt{x\left(a+bx^2+cx^4\right)}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)
```

output
```
Integral(sqrt(x*(a + b*x**2 + c*x**4))/sqrt(x), x)
```

3.107.7 Maxima [F]

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx = \int \frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

3.107.8 Giac [F]

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx = \int \frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx = \int \frac{\sqrt{c\,x^5+b\,x^3+a\,x}}{\sqrt{x}}\,dx$$

input
```
int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2),x)
```

output
```
int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2), x)
```

3.108 $\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx$

3.108.1 Optimal result

Integrand size = 24, antiderivative size = 194

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)*x^(1/2
)*(c*x^4+b*x^2+a)^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/4*b*arctanh(1/2*(2*c*x^2
+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(
c*x^5+b*x^3+a*x)^(1/2)+1/2*(c*x^5+b*x^3+a*x)^(1/2)/x^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+4\sqrt{a}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)-b\right.}{4\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

input
```
Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4] + 4*Sq
rt[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] - b
*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4*Sqrt[c]*Sqrt[x*
(a + b*x^2 + c*x^4)])
```

3.108.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1968, 2000, 1578, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}dx$$

$$\Big\downarrow \text{1968}$$

$$\frac{1}{2}\int \frac{bx^2+2a}{\sqrt{x}\sqrt{cx^5+bx^3+ax}}dx+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

$$\Big\downarrow \text{2000}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{bx^2+2a}{x\sqrt{cx^4+bx^2+a}}dx}{2\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{bx^2+2a}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{4\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(b\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2+2a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)}{4\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2b\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}+2a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)}{4\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+\frac{b\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}\right)}{4\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

$\downarrow$ 1154

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-4a\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}\right)}{4\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

$\downarrow$ 219

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)}{4\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

input
```
Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

output
```
Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) + (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(
-2*Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] + (b
*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c]))/(4*
Sqrt[a*x + b*x^3 + c*x^5])
```

3.108.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1968
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Simp[(n - q)*(p/(m + p*(2*n - q) + 1))   Int[x^(m + q)*
(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; Free
Q[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
 + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

rule 2000
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2
)*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fr
eeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && Pos
Q[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

3.108. $\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}} \, dx$

3.108.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\left(2\sqrt{a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)\sqrt{c}-b\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)-2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}\right)}{4\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}$	136

input
```
int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x*(c*x^4+b*x^2+a))^(1/2)*(2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b
*x^2+a)^(1/2))/x^2)*c^(1/2)-b*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1
/2)+b)/c^(1/2))-2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(
1/2)/c^(1/2)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 666, normalized size of antiderivative = 3.43

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\,ac)x}{x}\right)+2\,\sqrt{a}cx\log\left(-\frac{(b^2\cdots}{}\right.}{8\,cx}\right.$$

$$-\frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}(2\,cx^2+b)\sqrt{-c}\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)-\sqrt{a}cx\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\,cx}$$

input
```
integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x
)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(
-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b
*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))
/(c*x), -1/4*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 +
 b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - sqrt(a)*c*x*log(-((b^2
 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 +
 2*a)*sqrt(a)*sqrt(x))/x^5) - 2*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)
, 1/8*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*s
qrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + b*sqrt(c)*x*log(-(8*c^2*x^5
 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) +
 (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/4*(
2*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)
*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5
 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)
) + 2*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)]
```

3.108.6 Sympy [F]

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{x^{3/2}}\, dx = \int \frac{\sqrt{x\,(a + bx^2 + cx^4)}}{x^{\frac{3}{2}}}\, dx$$

input
```
integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)
```

output
```
Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)
```

3.108.7 Maxima [F]

$$\int \frac{\sqrt{ax + bx^3 + cx^5}}{x^{3/2}}\, dx = \int \frac{\sqrt{cx^5 + bx^3 + ax}}{x^{\frac{3}{2}}}\, dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)
```

3.108.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \int \frac{\sqrt{c\,x^5+b\,x^3+a\,x}}{x^{3/2}}\,dx$$

input
```
int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2),x)
```

output
```
int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2), x)
```

3.109 $\int x^{3/2}(ax+bx^3+cx^5)^{3/2}\,dx$

3.109.1 Optimal result

Integrand size = 24, antiderivative size = 244

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2-16ac)\,x^2)\sqrt{ax+bx^3+cx^5}}{640c^2}+\frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3b(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax+bx^3+cx^5}}$$

output
```
1/80*(8*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)*x^(1/2)/c-3/512*b*(-4*a*c+b^2)^
2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*
x^2+a)^(1/2)/c^(7/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/640*x^(3/2)*(b*(-4*a*c+5*b^
2)+4*c*(-16*a*c+5*b^2)*x^2)*(c*x^5+b*x^3+a*x)^(1/2)/c^2+1/1280*(128*a^2*c^
2-100*a*b^2*c+15*b^4)*(c*x^5+b*x^3+a*x)^(1/2)/c^3/x^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.74

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(15b^4-10b^3cx^2+128c^2\left(a+cx^4\right)^2+4b^2c(-25a+2\right.\right.}{2560c^{7/2}\sqrt{x\left(a+b\right.}}$$

input
```
Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^
4 - 10*b^3*c*x^2 + 128*c^2*(a + c*x^4)^2 + 4*b^2*c*(-25*a + 2*c*x^4) + 8*b
*c^2*x^2*(7*a + 22*c*x^4)) + 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x^2 - 2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4]]))/(2560*c^(7/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.109.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1966, 25, 1992, 25, 1996, 27, 1961, 1432, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$$

$$\Big\downarrow \ 1966$$

$$\frac{3\int -\sqrt{x}\left(\left(5b^2-16ac\right)x^2+2ab\right)\sqrt{cx^5+bx^3+ax}dx}{80c}+\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3\int \sqrt{x}\left(\left(5b^2-16ac\right)x^2+2ab\right)\sqrt{cx^5+bx^3+ax}dx}{80c}$$

$$\Big\downarrow \ 1992$$

$$\frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3\left(\frac{\int -\frac{x^{3/2}\left(\left(15b^4-100acb^2+128a^2c^2\right)x^2+2ab\left(5b^2-28ac\right)\right)}{\sqrt{cx^5+bx^3+ax}}dx}{24c}+\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}\right)}{80c}$$

$\downarrow$ 25

$$\frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3\left(\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}-\frac{\int \frac{x^{3/2}\left(\left(15b^4-100acb^2+128a^2c^2\right)x^2+2ab\left(5b^2-28ac\right)\right)}{\sqrt{cx^5+bx^3+ax}}dx}{24c}\right)}{80c}$$

$\downarrow$ 1996

$$\frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3\left(\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}-\frac{\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{2c\sqrt{x}}-\frac{\int \frac{15b\left(b^2-4ac\right)^2x^{3/2}}{\sqrt{cx^5+bx^3+ax}}dx}{2c}}{24c}\right)}{80c}$$

$\downarrow$ 27

$$\frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3\left(\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}-\frac{\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{2c\sqrt{x}}-\frac{15b\left(b^2-4ac\right)^2\int \frac{x^{3/2}}{\sqrt{cx^5+bx^3+ax}}dx}{2c}}{24c}\right)}{80c}$$

$\downarrow$ 1961

$$\frac{\sqrt{x}(3b+8cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-\frac{3\left(\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}-\frac{\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{2c\sqrt{x}}-\frac{15b\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\int \frac{x}{\sqrt{cx^4+bx^2+a}}d}{2c\sqrt{ax+bx^3+cx^5}}}{24c}\right.}{80c}$$

$\downarrow$ 1432

3.109. $\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$

$$\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-$$
$$\frac{3\left(\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}-\frac{\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{2c\sqrt{x}}-\frac{15b\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\int\frac{1}{\sqrt{cx^4+bx^2+a}}d}{4c\sqrt{ax+bx^3+cx^5}}}{24c}\right.}{80c}$$

$\Big\downarrow$ 1092

$$\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-$$
$$\frac{3\left(\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}-\frac{\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{2c\sqrt{x}}-\frac{15b\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\int\frac{1}{4c-x^4}d\frac{2cx}{\sqrt{cx^4+}}}{2c\sqrt{ax+bx^3+cx^5}}}{24c}\right.}{80c}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}-$$
$$\frac{3\left(\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{24c}-\frac{\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{2c\sqrt{x}}-\frac{15b\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\text{arctanh}\left(\frac{}{2\sqrt{c}}\right.}{4c^{3/2}\sqrt{ax+bx^3+cx^5}}}{24c}\right.}{80c}$$

input
```
Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
(Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(80*c) - (3*((x^(3/2
)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5]
)/(24*c) - (((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5
])/(2*c*Sqrt[x]) - (15*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*Sqrt
[a*x + b*x^3 + c*x^5]))/(24*c)))/(80*c)
```

3.109. $\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

3.109. $\int x^{3/2}(ax+bx^3+cx^5)^{3/2}\,dx$

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

3.109.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.83

method	result
risch	$\frac{(128c^4x^8+176c^3x^6b+256a\,c^3x^4+8b^2c^2x^4+56ab\,c^2x^2-10x^2c\,b^3+128a^2c^2-100a\,b^2c+15b^4)(c\,x^4+b\,x^2+a)\sqrt{x}}{1280c^3\sqrt{x(c\,x^4+b\,x^2+a)}} - \frac{3b(16a^2c^2-8a\,b^2c}{}$
default	$-\frac{\sqrt{x(c\,x^4+b\,x^2+a)}\left(-256c^{\frac{9}{2}}x^8\sqrt{c\,x^4+b\,x^2+a}-352b\,c^{\frac{7}{2}}x^6\sqrt{c\,x^4+b\,x^2+a}-512a\,c^{\frac{7}{2}}x^4\sqrt{c\,x^4+b\,x^2+a}-16b^2c^{\frac{5}{2}}x^4\sqrt{c\,x^4+b\,x^2+a}-11\right.}{}$

input
```
int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*(128*c^4*x^8+176*b*c^3*x^6+256*a*c^3*x^4+8*b^2*c^2*x^4+56*a*b*c^2*x
^2-10*b^3*c*x^2+128*a^2*c^2-100*a*b^2*c+15*b^4)*(c*x^4+b*x^2+a)/c^3*x^(1/2
)/(x*(c*x^4+b*x^2+a))^(1/2)-3/512*b*(16*a^2*c^2-8*a*b^2*c+b^4)/c^(7/2)*ln(
(1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*(c*x^4+b*x^2+a)^(1/2)*x^(1/2)
/(x*(c*x^4+b*x^2+a))^(1/2)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.62

$$\int x^{3/2}(ax+bx^3+cx^5)^{3/2}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3-4\,\sqrt{cx^5+bx^3+ax}(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\,ac)x}{x}\right)+4\,(1\right.$$

input
```
integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8
*b*c*x^3 - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^
2 + 4*a*c)*x)/x) + 4*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c
^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)
*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x), 1/2560*(15*(b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x
^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*(128*c^5*x^8 + 1
76*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*
c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(
x))/(c^4*x)]
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int x^{3/2}(ax+bx^3+cx^5)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int \left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(210) = 420.

Time = 0.45 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.62

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3\left(b^3-4abc\right)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx\right.\right.\right.}{c^{\frac{5}{2}}}\right.$$
$$+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3\left(5b^4-24ab^2c+\right.}{}\right.$$
$$+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-}{}\right.\right.$$

input
```
integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2
) - 3*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sq
rt(c) + b))/c^(5/2) + (3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*lo
g(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2)
)/c^(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 -
(5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a
*b^2*c + 16*a^2*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) + b))/c^(7/2) - (15*b^4*log(abs(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*
log(abs(b - 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c)
)) + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))/c^(7/2))*b + 1/7680*(
2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c
^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c
+ 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(2*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(9/2) + (105*b^5*log(
abs(b - 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(b - 2*sqrt(a)*sqrt(c)))
+ 720*a^2*b*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*sqrt(c)
- 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))/c^(9/2))*c
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx=\int x^{3/2}\,\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}\,dx$$

input
```
int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
```

output
```
int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.110 $\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx$

3.110.1 Optimal result

Integrand size = 24, antiderivative size = 487

$$\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx = \frac{(8b^4-57ab^2c+84a^2c^2)\,x^{3/2}(a+bx^2+cx^4)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt{x}(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{(3b+7cx^2)\,(ax+bx^3+cx^5)^{3/2}}{63c\sqrt{x}}$$
$$-\frac{\sqrt[4]{a}(8b^4-57ab^2c+84a^2c^2)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt[4]{a}\left(8b^4-57ab^2c+84a^2c^2+4\sqrt{a}b\sqrt{c}(b^2-6ac)\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

output
```
1/63*(7*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(1/2)+1/315*(84*a^2*c^2-57*
a*b^2*c+8*b^4)*x^(3/2)*(c*x^4+b*x^2+a)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^
5+b*x^3+a*x)^(1/2)-1/315*(b*(-9*a*c+4*b^2)+6*c*(-7*a*c+2*b^2)*x^2)*x^(1/2)
*(c*x^5+b*x^3+a*x)^(1/2)/c^2-1/315*a^(1/4)*(84*a^2*c^2-57*a*b^2*c+8*b^4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2
))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)
^(1/2)/c^(11/4)/(c*x^5+b*x^3+a*x)^(1/2)+1/630*a^(1/4)*(cos(2*arctan(c^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*(8*b^4-57*a*b^2*c+84*a^2*c^2+4*b*(-6*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/
2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^5+b*x^3+a
*x)^(1/2)
```

3.110.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.25

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx = \frac{\sqrt{x}\left(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-4b^4x^2-b^3cx^4+53b^2c^2x^6+85bc^3x^8+35c^4x^{10}+a^2c(24b+77cx^2)+c\right.}{}$$

input
```
Integrate[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
(Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 +
53*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*
(-4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b
^2*c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8
*b^5 + 65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*S
qrt[b^2 - 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/
(1260*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.110.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1966, 25, 1992, 25, 2000, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2} dx$$

$$\Big\downarrow \text{1966}$$

$$\frac{\int -\frac{\left(2\left(2b^2-7ac\right)x^2+ab\right)\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}dx}{21c}+\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}-\frac{\int \frac{\left(2\left(2b^2-7ac\right)x^2+ab\right)\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}dx}{21c}$$

$$\Big\downarrow \text{1992}$$

$$\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}-\frac{\frac{\int -\frac{\sqrt{x}\left(\left(8b^4-57acb^2+84a^2c^2\right)x^2+4ab\left(b^2-6ac\right)\right)}{\sqrt{cx^5+bx^3+ax}}dx}{15c}+\frac{\sqrt{x}\sqrt{ax+bx^3+cx^5}\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)}{15c}}{21c}$$

$\downarrow$ 25

$$\frac{(3b+7cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}-\frac{\frac{\sqrt{x}\left(6cx^2(2b^2-7ac)+b(4b^2-9ac)\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\int\frac{\sqrt{x}\left(\left(8b^4-57acb^2+84a^2c^2\right)x^2+4ab\left(b^2-6ac\right)\right)}{\sqrt{cx^5+bx^3+ax}}dx}{15c}}{21c}$$

$\downarrow$ 2000

$$\frac{(3b+7cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}-\frac{\frac{\sqrt{x}\left(6cx^2(2b^2-7ac)+b(4b^2-9ac)\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int\frac{\left(8b^4-57acb^2+84a^2c^2\right)x^2+4ab\left(b^2-6ac\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c\sqrt{ax+bx^3+cx^5}}}{21c}$$

$\downarrow$ 1511

$$\frac{(3b+7cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}-$$
$$\frac{\frac{\sqrt{x}\left(6cx^2(2b^2-7ac)+b(4b^2-9ac)\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(84a^2c^2 \ldots}{15c\sqrt{ax+bx^3+cx^5}}}{21c}$$

$\downarrow$ 27

$$\frac{(3b+7cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}-$$
$$\frac{\frac{\sqrt{x}\left(6cx^2(2b^2-7ac)+b(4b^2-9ac)\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(84a^2c^2-5 \ldots}{15c\sqrt{ax+bx^3+cx^5}}}{21c}$$

$\downarrow$ 1416

$$\frac{(3b+7cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}-$$
$$\frac{\frac{\sqrt{x}\left(6cx^2(2b^2-7ac)+b(4b^2-9ac)\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{E} \ldots}{2c^{3/4}\sqrt{a+bx^2+cx^4}} \ldots}{15c\sqrt{ax+bx^3 \ldots}}}{21c}$$

$\downarrow$ 1509

3.110. $\int\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$

$$\frac{(3b+7cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}} - \frac{\frac{\sqrt{x}\left(6cx^2(2b^2-7ac)+b(4b^2-9ac)\right)\sqrt{ax+bx^3+cx^5}}{15c} - \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E}{2c^{3/4}\sqrt{a+bx^2+cx^4}}\right.}{}}{}$$

input
```
Int[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
((3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(63*c*Sqrt[x]) - ((Sqrt[x]*(
b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1
5*c) - (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(-(((8*b^4 - 57*a*b^2*c + 84*a^2*c
^2)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(
1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c +
84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 +
 c*x^4])))/(15*c*Sqrt[a*x + b*x^3 + c*x^5]))/(21*c)
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

3.110. $\int \sqrt{x}(ax + bx^3 + cx^5)^{3/2}\, dx$

rule 2000
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2
)*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fr
eeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && Pos
Q[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

3.110.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.34

method	result
risch	$\frac{x^{\frac{3}{2}}\left(35c^{3}x^{6}+50b\,c^{2}x^{4}+77a\,c^{2}x^{2}+3b^{2}c\,x^{2}+24abc-4b^{3}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}{315c^{2}\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}-\frac{\left(-\frac{a\,b^{3}\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x}{a}}}{\sqrt{\frac{-b+\sqrt{-4ac+b}}{a}}}\right.}{}$
default	Expression too large to display

input
```
int((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x^(3/2)*(35*c^3*x^6+50*b*c^2*x^4+77*a*c^2*x^2+3*b^2*c*x^2+24*a*b*c-4
*b^3)/c^2*(c*x^4+b*x^2+a)/(x*(c*x^4+b*x^2+a))^(1/2)-1/315/c^2*(-a*b^3*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellip
ticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))+6*c*b*a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(84*
a^2*c^2-57*a*b^2*c+8*b^4)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))*(c*x^4+b*x^2+a)^(1/2)*x^(1/2)/(x*
(c*x^4+b*x^2+a))^(1/2)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00

$$\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left((8\,b^4c-57\,ab^2c^2+84\,a^2c^3)x^2\sqrt{\frac{b^2-4\,ac}{c^2}}-(8\,b^5-57\,ab^3c+84\,a^2bc^2)x^2\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\cdots}{\cdots}$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="fricas")
```

output
```
1/630*(sqrt(1/2)*((8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3)*x^2*sqrt((b^2 - 4*
a*c)/c^2) - (8*b^5 - 57*a*b^3*c + 84*a^2*b*c^2)*x^2)*sqrt(c)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 -
 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(
a*c)) - sqrt(1/2)*((8*b^4*c + 12*(7*a^2 + 2*a*b)*c^3 - (57*a*b^2 + 4*b^3)*
c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (8*b^5 + 12*(7*a^2*b - 2*a*b^2)*c^2 - (
57*a*b^3 - 4*b^4)*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*
elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/
2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(35*c^5*x^8 + 50*
b*c^4*x^6 + 8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3 + (3*b^2*c^3 + 77*a*c^4)*x
^4 - 4*(b^3*c^2 - 6*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*
x^2)
```

3.110.6 Sympy [F]

$$\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx = \int \sqrt{x}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)
```

output
```
Integral(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

3.110.7 Maxima [F]

$$\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx=\int (cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}\,dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

3.110.8 Giac [F]

$$\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx=\int (cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}\,dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(ax+bx^3+cx^5)^{3/2}\,dx=\int \sqrt{x}\,\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}\,dx$$

input
```
int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
```

output
```
int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.111 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx$

3.111.1 Optimal result

Integrand size = 24, antiderivative size = 177

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = -\frac{3(b^2-4ac)(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} + \frac{3(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx^4}\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}$$

output
```
1/16*(2*c*x^2+b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(3/2)+3/256*(-4*a*c+b^2)^2*ar
ctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+
a)^(1/2)/c^(5/2)/(c*x^5+b*x^3+a*x)^(1/2)-3/128*(-4*a*c+b^2)*(2*c*x^2+b)*(c
*x^5+b*x^3+a*x)^(1/2)/c^2/x^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.85

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \frac{(x(a+bx^2+cx^4))^{3/2}\left(\frac{\sqrt{c}(b+2cx^2)(-3b^2+8bcx^2+4c(5a+2cx^4))}{a+bx^2+cx^4} + \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{\quad}{-\sqrt{\ldots}}\right.}{(a+bx^2+cx^4)^{3\ldots}}\right.}{128c^{5/2}x^{3/2}}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

output
```
((x*(a + b*x^2 + c*x^4))^(3/2)*((Sqrt[c]*(b + 2*c*x^2)*(-3*b^2 + 8*b*c*x^2
 + 4*c*(5*a + 2*c*x^4)))/(a + b*x^2 + c*x^4) + (3*(b^2 - 4*a*c)^2*ArcTanh[
(Sqrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^2 + c*x^4])])/(a + b*x^2 + c*x^4)^(
3/2)))/(128*c^(5/2)*x^(3/2))
```

3.111.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1965, 1965, 1961, 1432, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1965}$$

$$\frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}-\frac{3\left(b^2-4ac\right)\int \sqrt{x}\sqrt{cx^5+bx^3+ax}dx}{16c}$$

$$\Big\downarrow \text{1965}$$

$$\frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\left(b^2-4ac\right)\int \frac{x^{3/2}}{\sqrt{cx^5+bx^3+ax}}dx}{8c}\right)}{16c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\int \frac{x}{\sqrt{cx^4+bx^2+a}}dx}{8c\sqrt{ax+bx^3+cx^5}}\right)}{16c}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{16c\sqrt{ax+bx^3+cx^5}}\right)}{16c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(b+2cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}(b^2-4ac)\sqrt{a+bx^2+cx^4}\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{8c\sqrt{ax+bx^3+cx^5}}\right)}{16c}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx^2)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}(b^2-4ac)\sqrt{a+bx^2+cx^4}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}\right)}{16c}$$

input
```
Int[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

output
```
((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x^(3/2)) - (3*(b^2 - 4*a
*c)*(((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a
*c)*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])))/(16*c)
```

3.111.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

3.111. $\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx$

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

3.111.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

method	result
risch	$\frac{\left(16c^{3}x^{6}+24b\,c^{2}x^{4}+40a\,c^{2}x^{2}+2b^{2}c\,x^{2}+20abc-3b^{3}\right)\left(c\,x^{4}+b\,x^{2}+a\right)\sqrt{x}}{128c^{2}\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}+\frac{3\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)\sqrt{c\,x^{4}}}{256c^{\frac{5}{2}}\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}$
default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\left(32c^{\frac{7}{2}}x^{6}\sqrt{c\,x^{4}+b\,x^{2}+a}+48b\,c^{\frac{5}{2}}x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}+80a\,c^{\frac{5}{2}}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}+4b^{2}c^{\frac{3}{2}}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}+48\ln\left(\frac{2c\,x^{2}}{}\right.\right.}{}$

input
```
int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(16*c^3*x^6+24*b*c^2*x^4+40*a*c^2*x^2+2*b^2*c*x^2+20*a*b*c-3*b^3)*(c
*x^4+b*x^2+a)/c^2*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)+3/256*(16*a^2*c^2-8*a*
b^2*c+b^4)/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*(c*x^4+
b*x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.88

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\right.}{x}\right.$$

$$-\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}(2\,cx^2+b)\sqrt{-c}\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)-2\,(16\,c^4x^6+24\,bc^3x^4-3\,b^3c+20\,abc^2}{256\,c^3x}$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c
*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 +
4*a*c)*x)/x) + 4*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^
2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x), -1/256*
(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3
 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - 2*(1
6*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x
^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x)]
```

3.111.6 Sympy [F]

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(x(a+bx^2+cx^4))^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

output
```
Integral((x*(a + b*x**2 + c*x**4))**(3/2)/sqrt(x), x)
```

3.111.7 Maxima [F]

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(cx^5 + bx^3 + ax)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x), x)
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(149) = 298.

Time = 0.45 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.81

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{\sqrt{x}}\,dx = \frac{1}{16}\left(2\sqrt{cx^4 + bx^2 + a}\left(2x^2 + \frac{b}{c}\right) + \frac{(b^2 - 4ac)\log\left(\left|2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + \cdots}\right.\right.\right.}{c^{\frac{3}{2}}}\right.$$
$$+\frac{1}{96}\left(2\sqrt{cx^4 + bx^2 + a}\left(2\left(4x^2 + \frac{b}{c}\right)x^2 - \frac{3b^2 - 8ac}{c^2}\right) - \frac{3(b^3 - 4abc)\log\left(\left|2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\cdots\right.\right.}{c^{\frac{5}{2}}}\right.$$
$$+\frac{1}{768}\left(2\sqrt{cx^4 + bx^2 + a}\left(2\left(4\left(6x^2 + \frac{b}{c}\right)x^2 - \frac{5b^2c - 12ac^2}{c^3}\right)x^2 + \frac{15b^3 - 52abc}{c^3}\right) + \frac{3(5b^4 - 24ab^2c + \cdots}{\cdots}\right.$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2) - (b^2*log(abs
(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sqrt(
a)*b*sqrt(c))/c^(3/2))*a + 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c
)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(5/2) + (3*b^3*log(abs(b - 2*sqrt
(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*s
qrt(c) - 16*a^(3/2)*c^(3/2))/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)
*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*
b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(7/2) - (15*b^4*log(abs(b - 2*sqrt
(a)*sqrt(c))) - 72*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*lo
g(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(
3/2))/c^(7/2))*c
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(c\,x^5+b\,x^3+a\,x)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2),x)
```

output
```
int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2), x)
```

3.112 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx$

3.112.1 Optimal result

Integrand size = 24, antiderivative size = 425

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx = -\frac{2b(b^2-8ac)\,x^{3/2}(a+bx^2+cx^4)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt{x}(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c}+\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$
$$+\frac{2\sqrt[4]{a}b(b^2-8ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}(b^2-20ac)+2b(b^2-8ac)\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

output
```
1/7*(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2)-2/35*b*(-8*a*c+b^2)*x^(3/2)*(c*x^4+b*x
^2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+1/35*(3*b*c*x^
2+10*a*c+b^2)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+2/35*a^(1/4)*b*(-8*a*c+b^2
)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4
)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/70*a^(1/4)*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^
(1/2))*(2*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*
x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.27

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{x^{3/2}} dx = \frac{\sqrt{x}\left(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(15a^2c + a(b^2 + 23bcx^2 + 20c^2x^4) + x^2(b^3 + 9b^2cx^2 + 13bc\right.}{}$$

input
```
Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

output
```
(Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*
c*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6))
- I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x
^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-
b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 -
4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[x*(a + b*x^2 + c*x^4)])
```

3.112.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1968, 1992, 25, 2000, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax + bx^3 + cx^5)^{3/2}}{x^{3/2}} dx$$

$$\Big\downarrow \text{1968}$$

$$\frac{3}{7}\int \frac{(bx^2 + 2a)\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}}dx + \frac{(ax + bx^3 + cx^5)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow 1992$$

$$\frac{3}{7}\left(\frac{\int -\frac{\sqrt{x}(2b(b^2-8ac)x^2+a(b^2-20ac))}{\sqrt{cx^5+bx^3+ax}}dx}{15c}+\frac{\sqrt{x}\sqrt{ax+bx^3+cx^5}(10ac+b^2+3bcx^2)}{15c}\right)+\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow 25$$

$$\frac{3}{7}\left(\frac{\sqrt{x}(10ac+b^2+3bcx^2)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\int \frac{\sqrt{x}(2b(b^2-8ac)x^2+a(b^2-20ac))}{\sqrt{cx^5+bx^3+ax}}dx}{15c}\right)+\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow 2000$$

$$\frac{3}{7}\left(\frac{\sqrt{x}(10ac+b^2+3bcx^2)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{2b(b^2-8ac)x^2+a(b^2-20ac)}{\sqrt{cx^4+bx^2+a}}dx}{15c\sqrt{ax+bx^3+cx^5}}\right)+\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow 1511$$

$$\frac{3}{7}\left(\frac{\sqrt{x}(10ac+b^2+3bcx^2)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2}}\right.}{15c\sqrt{ax+bx^3+cx^5}}\right.$$

$$\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow 27$$

$$\frac{3}{7}\left(\frac{\sqrt{x}(10ac+b^2+3bcx^2)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2}}\right.}{15c\sqrt{ax+bx^3+cx^5}}\right.$$

$$\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow 1416$$

3.112. $\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx$

$$\frac{3}{7}\left(\frac{\sqrt{x}\left(10ac+b^2+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx}{(\sqrt{a}+}}}{2\sqrt[4]{c}\sqrt{a+b}}\right.}{15c\sqrt{ax}}\right.$$

$$\frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{3}{7}\left(\frac{\sqrt{x}\left(10ac+b^2+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx}{(\sqrt{a}+}}}{2\sqrt[4]{c}\sqrt{a+b}}\right.}{}\right.$$

$$\frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}}$$

input
```
Int[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

3.112. $\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{x^{3/2}}\,dx$

output
```
(a*x + b*x^3 + c*x^5)^(3/2)/(7*Sqrt[x]) + (3*((Sqrt[x]*(b^2 + 10*a*c + 3*b
*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(15*c) - (Sqrt[x]*Sqrt[a + b*x^2 + c*x^
4]*((-2*b*(b^2 - 8*a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*
x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sq
rt[a]*(b^2 - 20*a*c) + (2*b*(b^2 - 8*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x
^2 + c*x^4])))/(15*c*Sqrt[a*x + b*x^3 + c*x^5])))/7
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1968
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Simp[(n - q)*(p/(m + p*(2*n - q) + 1))   Int[x^(m + q)*
(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; Free
Q[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
 + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

rule 2000
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2
)*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fr
eeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && Pos
Q[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

3.112. $\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx$

3.112.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.46

method	result
risch	$\frac{x^{\frac{3}{2}}\left(5c^2x^4+8bc\,x^2+15ac+b^2\right)\left(c\,x^4+b\,x^2+a\right)}{35c\sqrt{x\left(c\,x^4+b\,x^2+a\right)}}+\frac{\left(-\frac{b^2a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\right.}{}$
default	Expression too large to display

input
```
int((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x^(3/2)*(5*c^2*x^4+8*b*c*x^2+15*a*c+b^2)/c*(c*x^4+b*x^2+a)/(x*(c*x^4+
b*x^2+a))^(1/2)+1/35/c*(-1/4*b^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+5*c*a^2*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(16*a*b*c-2*b^3)*a*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*
(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))*(c*x^4+b*
x^2+a)^(1/2)*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.96

$$\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{x^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left(\left(b^3c-8\,abc^2\right)x^2\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(b^4-8\,ab^2c\right)x^2\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}}{2}\right.}{}$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
-1/70*(2*sqrt(1/2)*((b^3*c - 8*a*b*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (b^4
 - 8*a*b^2*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ellipti
c_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*
sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b^3*c + 20*a
*c^3 - (16*a*b + b^2)*c^2)*x^2*sqrt((b^2 - 4*a*c)/c^2) - (2*b^4 - 20*a*b*c
^2 - (16*a*b^2 - b^3)*c)*x^2)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)
/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x)
, 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(5*c^4*x^6 +
8*b*c^3*x^4 - 2*b^3*c + 16*a*b*c^2 + (b^2*c^2 + 15*a*c^3)*x^2)*sqrt(c*x^5
+ b*x^3 + a*x)*sqrt(x))/(c^3*x^2)
```

3.112.6 Sympy [F]

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx = \int \frac{(x(a+bx^2+cx^4))^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)
```

output
```
Integral((x*(a + b*x**2 + c*x**4))**(3/2)/x**(3/2), x)
```

3.112.7 Maxima [F]

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx = \int \frac{(cx^5+bx^3+ax)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

3.112.8 Giac [F]

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx=\int \frac{(cx^5+bx^3+ax)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx=\int \frac{(c\,x^5+b\,x^3+a\,x)^{3/2}}{x^{3/2}}\,dx$$

input
```
int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x)
```

output
```
int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2), x)
```

3.113 $\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx$

3.113.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx=\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

output
```
1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+
b*x^2+a)^(1/2)/c^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx=-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

input
```
Integrate[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
-1/2*(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4]])/(Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.113.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1961, 1432, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx$$

$$\Big\downarrow \text{1961}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{x}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

3.113.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

3.113.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\ln\left(\frac{2 c\,x^{2}+2 \sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{2 \sqrt{c}}\right)}{2 \sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}$	72

input
```
int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/2*(2*c*x^
2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))/c^(1/2)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \left[\frac{\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4\,ac)x}{x}\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{-c}\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)}{2\,c}\right]$$

input
```
integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 +
b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2
*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x
^3 + a*c*x))/c]
```

3.113.6 Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

input
```
integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.113.7 Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{2\sqrt{c}} + \frac{\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)}{2\sqrt{c}}$$

input
```
integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/sqrt(
c) + 1/2*log(abs(b - 2*sqrt(a)*sqrt(c)))/sqrt(c)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x^{3/2}}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

input
```
int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.114 $\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx$

3.114.1 Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

output
```
1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1
/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))
^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = -\frac{i\sqrt{x}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

input
```
Integrate[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
((-I)*Sqrt[x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 +
 c*x^4)])
```

3.114.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1961, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx$$

$$\Big\downarrow \text{ 1961}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

3.114.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

3.114.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

method	result	s
default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\sqrt{-\frac{2\left(\sqrt{-4ac+b^{2}}\,x^{2}-b\,x^{2}-2a\right)}{a}}\,\sqrt{\frac{\sqrt{-4ac+b^{2}}\,x^{2}+b\,x^{2}+2a}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{2}\,\sqrt{\frac{b\sqrt{-4ac+b^{2}}-2ac+b^{2}}{ac}}}{2}\right)}{2\sqrt{x}\,\left(c\,x^{4}+b\,x^{2}+a\right)\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}$	

input
```
int(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(-2*((-4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*
c+b^2)^(1/2)*x^2+b*x^2+2*a))^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2
))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx$$
$$= \frac{\sqrt{\frac{1}{2}}\left(c\sqrt{\frac{b^2-4\,ac}{c^2}}+b\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}F(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})}{2\,a\sqrt{c}}$$

input
```
integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*(c*sqrt((b^2 - 4*a*c)/c^2) + b)*sqrt((c*sqrt((b^2 - 4*a*c)/c
^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) -
b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))/(a*sqrt(c
))
```

3.114.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

input
```
integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.114.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.114.8 Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{\sqrt{x}}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.115 $\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$

3.115.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1
/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx=\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c
*x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.115.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1960, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$$

$$\Big\downarrow \ 1960$$

$$-\int \frac{1}{4a-\frac{x(bx^2+2a)^2}{cx^5+bx^3+ax}}d\frac{\sqrt{x}(bx^2+2a)}{\sqrt{cx^5+bx^3+ax}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])
]/Sqrt[a]
```

3.115.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1960
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[-2/(n - q)   Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*
((2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m
, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[m, q/2 - 1]
```

3.115.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{a}}$	72

input
```
int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2
*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.69

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a}\right]$$

input
```
integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3
+ a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/
2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*
x^3 + a^2*x))/a]
```

3.115.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{1}{\sqrt{x}\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

input
```
integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

3.115.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{1}{\sqrt{cx^5+bx^3+ax}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx=\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arcta
n(sqrt(a)/sqrt(-a))/sqrt(-a)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

3.116 $\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$

3.116.1 Optimal result

Integrand size = 24, antiderivative size = 330

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx &= \frac{\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}\\ &-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}}\\ &+\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}\end{aligned}$$

output
```
x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^
(1/2)-(c*x^5+b*x^3+a*x)^(1/2)/a/x^(3/2)-c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*
x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x
^3+a*x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.116.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \frac{-4(a+bx^2+cx^4)+\frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(E\left(i\text{arcsinh}\left(\right.\right.\right.}{}}{4a\sqrt{x}\sqrt{x\,(a+b}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

output
```
(-4*(a + b*x^2 + c*x^4) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*x*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqr
t[x]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.116.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1976, 27, 1961, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$$

$$\Big\downarrow \text{1976}$$

$$\frac{\int \frac{cx^{5/2}}{\sqrt{cx^5+bx^3+ax}}dx}{a}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{x^{5/2}}{\sqrt{cx^5+bx^3+ax}}dx}{a}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}$$

$$\Big\downarrow \text{1961}$$

$$\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}dx}{a\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}$$

$\downarrow$ 1459

$$\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}$$

$\downarrow$ 27

$$\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}$$

$\downarrow$ 1416

$$\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}$$

$\downarrow$ 1509

$$\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E(}{\sqrt[4]{c}\sqrt{a+b}}\right.}{a\sqrt{ax+bx^3+cx^5}}$$

$$\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}$$

input
```
Int[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

3.116. $\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$

output
```
-(Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2))) + (c*Sqrt[x]*Sqrt[a + b*x^2 + c*x
^4]*(-((-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b
*x^2 + c*x^4])))/(a*Sqrt[a*x + b*x^3 + c*x^5])
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1976
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
/(a*(m + p*q + 1))), x] - Simp[1/(a*(m + p*q + 1))   Int[x^(m + n - q)*(b*(
m + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
 q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && Eq
Q[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0
]
```

3.116.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{c\,x^{4}+b\,x^{2}+a}{a\sqrt{x}\,\sqrt{x(c\,x^{4}+b\,x^{2}+a)}}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\left(F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4a}\right.}{ac}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{x(c\,x^{4}}}$
default	$\frac{\left(-\sqrt{-4ac+b^{2}}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,c\,x^{4}-\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,bc\,x^{4}-c\sqrt{-\frac{2\left(\sqrt{-4ac+b^{2}}\,x^{2}-b\,x^{2}-2a\right)}{a}}\,\sqrt{\frac{\sqrt{-4ac+b^{2}}\,x^{2}+b\,x^{2}+2a}{a}}\,axF\left(\frac{x\sqrt{2}\,\sqrt{}}{}\right.\right.}{}$

input
```
int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+b*x^2+a)/a/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)-1/2*c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))*x^(1/2)/(x*(c*x^4+b*x^2+a
))^(1/2)
```

3.116.5 Fricas [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c*x^7 + b*x^5 + a*x^3), x)
```

3.116.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

input
```
integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

3.116.7 Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

3.116.8 Giac [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

3.117 $\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx$

3.117.1 Optimal result

Integrand size = 24, antiderivative size = 391

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{x^{3/2}(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}}$$

output
```
x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)-b*x^(3/
2)*(c*x^4+b*x^2+a)*c^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x
^3+a*x)^(1/2)+b*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)
/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/
2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/
2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/
2)
```

3.117.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.66 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.18

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx =$$

$$-\frac{\sqrt{x}\left(-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b^2-2ac+bcx^2)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcs}\right.\right.}{}$$

input
```
Integrate[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
-1/4*(Sqrt[x]*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2
) + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c
+ b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(a*(b^2 - 4*a*c)*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.117.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1971, 27, 2000, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}dx$$

$$\Big\downarrow \; 1971$$

$$\frac{x^{3/2}(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}}-\frac{\int \frac{c\sqrt{x}(bx^2+2a)}{\sqrt{cx^5+bx^3+ax}}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{c\int \frac{\sqrt{x}\left(bx^2+2a\right)}{\sqrt{cx^5+bx^3+ax}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 2000$$

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \ 1511$$

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \ 1416$$

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \ 1509$$

3.117. $\int \frac{x^{3/2}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \frac{b\left(\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\cdots}\right.}{\cdots}\right.}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
(x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^
5]) - (c*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(-((b*(-((x*Sqrt[a + b*x^2 + c*x^
4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))
/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*
x^4])))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5])
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 2000
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2
)*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fr
eeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && Pos
Q[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

3.117.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.36

method	result
default	$\frac{\sqrt{x(c x^4+b x^2+a)}\left(-\sqrt{-4ac+b^2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,bc x^3-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,b^2 c x^3+c\sqrt{-\frac{2\left(\sqrt{-4ac+b^2}\,x^2-b x^2-2a\right)}{a}}\,\sqrt{\frac{\sqrt{-4ac+b^2}\,x^2+\cdots}{a}}\cdots\right.}{\cdots}$

3.117. $\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx$

input
```
int(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)*(-(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*b*c*x^3-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*c*x^3+c*(-2
*((-4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+
b*x^2+2*a))^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*(-4*a*c+b^2)
^(1/2)+b*c*(-2*((-4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b
^2)^(1/2)*x^2+b*x^2+2*a))^(1/2)*a*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2)
)+2*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*c*x-(-4*a*c+b^2
)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*x+2*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*a*b*c*x-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3*x)/(c*x^4+b*x^2+a)
/a/(4*a*c-b^2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.23

$$\int \frac{x^{3/2}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(b^2cx^6+b^3x^4+ab^2x^2-\left(bc^2x^6+b^2cx^4+abcx^2\right)\sqrt{\frac{b^2-4\,ac}{c^2}}\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{}$$

input
```
integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*(b^2*c*x^6 + b^3*x^4 + a*b^2*x^2 - (b*c^2*x^6 + b^2*c*x^4 +
 a*b*c*x^2)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^
2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b
)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2
)*((b^2*c + 2*b*c^2)*x^6 + (b^3 + 2*b^2*c)*x^4 + (a*b^2 + 2*a*b*c)*x^2 - (
(b*c^2 - 2*c^3)*x^6 + (b^2*c - 2*b*c^2)*x^4 + (a*b*c - 2*a*c^2)*x^2)*sqrt(
(b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ellipt
ic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c
*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*sqrt(c*x^5 + b*x^3 + a*
x)*(2*a*c^2*x^2 + a*b*c)*sqrt(x))/((a*b^2*c^2 - 4*a^2*c^3)*x^6 + (a*b^3*c
- 4*a^2*b*c^2)*x^4 + (a^2*b^2*c - 4*a^3*c^2)*x^2)
```

3.117.6 Sympy [F]

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(x\,(a+bx^2+cx^4))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

output
```
Integral(x**(3/2)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

3.117.7 Maxima [F]

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^5+bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

3.117.8 Giac [F]

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^5+bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{x^{3/2}}{{\left(c\,x^5+b\,x^3+a\,x\right)}^{3/2}} \,d x$$

input
```
int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
```

output
```
int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.118 $\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx$

3.118.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx=\frac{\sqrt{x}(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(3
/2)+(b*c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx=-\frac{\sqrt{x}\left(\sqrt{a}(b^2-2ac+bcx^2)+(b^2-4ac)\sqrt{a+bx^2+cx^4}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)\right)}{a^{3/2}(-b^2+4ac)\sqrt{x(a+bx^2+cx^4)}}$$

input
```
Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
-((Sqrt[x]*(Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) + (b^2 - 4*a*c)*Sqrt[a + b*x^2
 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]))/(a^(3
/2)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)]))
```

3.118.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1969, 1960, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}dx$$

$$\Big\downarrow \text{1969}$$

$$\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^5+bx^3+ax}}dx}{a}+\frac{\sqrt{x}(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1960}$$

$$\frac{\sqrt{x}(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}}-\frac{\int \frac{1}{4a-\frac{x(bx^2+2a)^2}{cx^5+bx^3+ax}}d\frac{\sqrt{x}(bx^2+2a)}{\sqrt{cx^5+bx^3+ax}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x}(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{ax+bx^3+cx^5}}-\frac{\text{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

input
```
Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

output
```
(Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^
5]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5]
)]/(2*a^(3/2))
```

3.118.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1960
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[-2/(n - q)   Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*
((2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m
, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[m, q/2 - 1]
```

rule 1969
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r,
2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n,
0] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, (-(n - q))*(2*p
+ 3)]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(87) = 174$.

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{x(c x^4+b x^2+a)}\left(2bc x^2\sqrt{a}+4\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)ac\sqrt{c x^4+b x^2+a}-\ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)b^2\sqrt{c x^4+b x^2}\right.}{2a^{\frac{3}{2}}\sqrt{x}\,(c x^4+b x^2+a)(4ac-b^2)}$

input
```
int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x*(c*x^4+b*x^2+a))^(1/2)/a^(3/2)*(2*b*c*x^2*a^(1/2)+4*ln((2*a+b*x^2+
2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*c*(c*x^4+b*x^2+a)^(1/2)-ln((2*a+b*
x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^2*(c*x^4+b*x^2+a)^(1/2)-4*a^(3
/2)*c+2*b^2*a^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(87) = 174$.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 4.12

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^5+(b^3-4\,abc)x^3+(ab^2-4\,a^2c)x)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+\cdots}{\cdots}\right.}{4\left((a^2b^2c-4\,a^3c^2)x^5+(a^2b^3\cdots\right.}\right.$$

input
```
integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x
^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x
)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a
^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x), 1/2*(((b^2*c - 4*a*c^2)*
x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(
c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a
^2*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x)
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)]
```

3.118.6 Sympy [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{\sqrt{x}}{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

output
```
Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

3.118.7 Maxima [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(cx^5+bx^3+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(87) = 174$.

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{\frac{abcx^2}{a^2b^2-4a^3c}+\frac{ab^2-2a^2c}{a^2b^2-4a^3c}}{\sqrt{cx^4+bx^2+a}} - \frac{ab^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4a^2c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}b^2-2\sqrt{-a}a^{\frac{3}{2}}c}{\sqrt{-a}a^2b^2-4\sqrt{-a}a^3c} + \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

input
```
integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
(a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sq
rt(c*x^4 + b*x^2 + a) - (a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(s
qrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)/(sqrt(-a)*
a^2*b^2 - 4*sqrt(-a)*a^3*c) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{\sqrt{x}}{(c\,x^5+b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
```

output
```
int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.119 $\int \frac{1}{\sqrt{x}(ax+bx^3+cx^5)^{3/2}}\,dx$

3.119.1 Optimal result

Integrand size = 24, antiderivative size = 468

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx &= \frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\,\sqrt{x}\sqrt{ax+bx^3+cx^5}} \\ &+ \frac{2\sqrt{c}(b^2-3ac)\,x^{3/2}(a+bx^2+cx^4)}{a^2\,(b^2-4ac)\,\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2\,(b^2-4ac)\,x^{3/2}} \\ &- \frac{2\sqrt[4]{c}(b^2-3ac)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} \\ &+ \frac{\sqrt[4]{c}\left(2b^2+\sqrt{a}b\sqrt{c}-6ac\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}}\end{aligned}$$

output
```
2*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a^2/(-4*a*c+b^2)/(a^(1/2)+x
^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(
1/2)/(c*x^5+b*x^3+a*x)^(1/2)-2*(-3*a*c+b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-
4*a*c+b^2)/x^(3/2)-2*c^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)
*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2
)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*a*c+b^2)/(c*
x^5+b*x^3+a*x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4
))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*
a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a
^(7/4)/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.41 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(-4a^2c+2b^2x^2(b+cx^2)+a(b^2-7bcx^2-6c^2x^4))-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2}}{b+\sqrt{}}}}{}$$

input
```
Integrate[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

output
```
-1/2*(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2)
+ a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*
c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] -
 3*a*c*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^2*(b^2 - 4*a*c)*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.119.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1971, 25, 1998, 25, 27, 2000, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}}dx$$

$$\Big\downarrow \text{1971}$$

$$\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\int -\frac{bcx^2+2\left(b^2-3ac\right)}{x^{3/2}\sqrt{cx^5+bx^3+ax}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bcx^2+2\left(b^2-3ac\right)}{x^{3/2}\sqrt{cx^5+bx^3+ax}}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{\int -\frac{c\sqrt{x}\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^5+bx^3+ax}}dx}{a}-\frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{c\sqrt{x}\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^5+bx^3+ax}}dx}{a}-\frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{c\int \frac{\sqrt{x}\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^5+bx^3+ax}}dx}{a}-\frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{2000}$$

$$\frac{\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{2\left(b^2-3ac\right)x^2+ab}{\sqrt{cx^4+bx^2+a}}dx}{a\sqrt{ax+bx^3+cx^5}}-\frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}(b^2-3ac)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\sqrt{ax+bx^3+cx^5}}-\frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2(b^2-3ac)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\sqrt{ax+bx^3+cx^5}}-\frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \; 1416$$

$$\frac{\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\sqrt{ax+bx^3+cx^5}}}{a(b^2-4ac)}$$

$$\frac{-2ac+b^2+bcx^2}{a\sqrt{x}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \; 1509$$

$$\frac{\frac{c\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{a\sqrt{ax+bx^3+cx^5}}}{a(b^2-4ac)}$$

$$\frac{-2ac+b^2+bcx^2}{a\sqrt{x}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

3.119. $\int \frac{1}{\sqrt{x}(ax+bx^3+cx^5)^{3/2}}\,dx$

output
```
(b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]
) + ((-2*(b^2 - 3*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(a*x^(3/2)) + (c*Sqrt[x]
*Sqrt[a + b*x^2 + c*x^4]*((-2*(b^2 - 3*a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])
/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sq
rt[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^2 - 3*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a
+ b*x^2 + c*x^4])))/(a*Sqrt[a*x + b*x^3 + c*x^5]))/(a*(b^2 - 4*a*c))
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

rule 2000
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2
)*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fr
eeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && Pos
Q[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. 2(454) = 908.

Time = 4.44 (sec) , antiderivative size = 1136, normalized size of antiderivative = 2.43

method	result	size
default	Expression too large to display	1136
risch	Expression too large to display	1497

input
```
int(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x*(c*x^4+b*x^2+a))^(1/2)/x^(3/2)*(12*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*a*c^2*x^4-4*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*b^2*c*x^4+12*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b*c^2*x^4-4*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3*c*x^4+a*b*c*(-2*((-4*a*c+b^2)^(1/2)*
x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2+2*a))^(1/2)*Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4
*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x*(-4*a*c+b^2)^(1/2)+12*(-2*((-4*a*
c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2+2*
a))^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^
(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*c^2*x-3*a*b^2*c*(-
2*((-4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2
+b*x^2+2*a))^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x-12*(-2*((-4
*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^2
+2*a))^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*c^2*x+4*(-2*((-
4*a*c+b^2)^(1/2)*x^2-b*x^2-2*a)/a)^(1/2)*(1/a*((-4*a*c+b^2)^(1/2)*x^2+b*x^
2+2*a))^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^2*c*x+14*(-4*a
*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b*c*x^2-4*(-4*a*c+b^2...
```

3.119.5 Fricas [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{1}{(cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + (b^2 +
2*a*c)*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

3.119.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,(x\,(a+bx^2+cx^4))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

3.119.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{1}{(cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

3.119.8 Giac [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx = \int \frac{1}{(cx^5+bx^3+ax)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx=\int \frac{1}{\sqrt{x}\,(c\,x^5+b\,x^3+a\,x)^{3/2}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

3.120 $\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

3.120.1 Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}}+\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}$$

output
```
3/4*b*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(
5/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/
2*(-8*a*c+3*b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-4*a*c+b^2)/x^(5/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(-4a^2c+3b^2x^2\left(b+cx^2\right)+a\left(b^2-10bcx^2-8c^2x^4\right)\right)+3b\left(b^2-4ac\right)x^2\sqrt{}}{2a^{5/2}\left(-b^2+4ac\right)x^{3/2}\sqrt{x\left(a+bx^2+\right.}}$$

input
```
Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

output
```
(Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x
^4)) + 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2
- Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(5/2)*(-b^2 + 4*a*c)*x^(3/2)*Sqr
t[x*(a + b*x^2 + c*x^4)])
```

3.120.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1971, 25, 1998, 27, 1960, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(ax+bx^3+cx^5\right)^{3/2}}dx$$

$$\Big\downarrow \text{1971}$$

$$\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\int -\frac{3b^2+2cx^2b-8ac}{x^{5/2}\sqrt{cx^5+bx^3+ax}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3b^2+2cx^2b-8ac}{x^{5/2}\sqrt{cx^5+bx^3+ax}}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{\int \frac{3b\left(b^2-4ac\right)}{\sqrt{x}\sqrt{cx^5+bx^3+ax}}dx}{2a}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2ax^{5/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{x}\sqrt{cx^5+bx^3+ax}}dx}{2a}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2ax^{5/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{1960}$$

$$\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-\frac{x\left(bx^2+2a\right)^2}{cx^5+bx^3+ax}}d\frac{\sqrt{x}\left(bx^2+2a\right)}{\sqrt{cx^5+bx^3+ax}}}{2a}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2ax^{5/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{3/2}}-\frac{(3b^2-8ac)\sqrt{ax+bx^3+cx^5}}{2ax^{5/2}}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

output
```
(b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]
) + (-1/2*((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(a*x^(5/2)) + (3*b*(
b^2 - 4*a*c)*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 +
 c*x^5])])/(4*a^(3/2)))/(a*(b^2 - 4*a*c))
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1960
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[-2/(n - q)   Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*
((2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m
, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[m, q/2 - 1]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

3.120.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{x(c\,x^4+b\,x^2+a)}\left(-16a^{\frac{3}{2}}c^2x^4+6b^2c\,x^4\sqrt{a}+12\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)abc\,x^2\sqrt{c\,x^4+b\,x^2+a}-3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x}}{x^2}\right.\right.}{4a^{\frac{5}{2}}x^{\frac{5}{2}}(c\,x^4+b\,x^2+a)(4ac-b^2)}$
risch	$-\frac{c\,x^4+b\,x^2+a}{2a^2x^{\frac{3}{2}}\sqrt{x(c\,x^4+b\,x^2+a)}}+\frac{\left(\frac{b^2c\,x^2}{a^2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}+\frac{b^3}{4a^2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{2c^2x^2}{a\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b}{4a^2\sqrt{c\,x^4+b\,x^2+a}}\right.}{\sqrt{x(c\,x^4+b\,x^2+a)}}$

input
```
int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x*(c*x^4+b*x^2+a))^(1/2)/a^(5/2)*(-16*a^(3/2)*c^2*x^4+6*b^2*c*x^4*a^(
1/2)+12*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*b*c*x^2*(c*x
^4+b*x^2+a)^(1/2)-3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^
3*x^2*(c*x^4+b*x^2+a)^(1/2)-20*a^(3/2)*b*c*x^2+6*a^(1/2)*b^3*x^2-8*a^(5/2)
*c+2*a^(3/2)*b^2)/x^(5/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.30

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^7+(b^4-4\,ab^2c)x^5+(ab^3-4\,a^2bc)x^3\right)\sqrt{a}\log\left(-\frac{(b^2+4\,ac}{}\right.}{8\left((a\right.}\right.$$

input
```
integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*
b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x + 4*sqrt(c
*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) - 4*sqrt(c*x^5 + b
*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 -
10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*
b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b
^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*
x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2
*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2
- 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*
x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)]
```

3.120.6 Sympy [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

3.120.7 Maxima [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

3.120.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx=\int \frac{1}{x^{3/2}\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}}\,dx$$

input
```
int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
```

output
```
int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

3.121 $\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx$

3.121.1 Optimal result

Integrand size = 34, antiderivative size = 51

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx=-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

output
```
-2*x^(-1/2+1/2*n)*(2*c*x+b)/(-4*a*c+b^2)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(1/2
)
```

3.121.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx=-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{x^{-1+n}(a+x(b+cx))}}$$

input
```
Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

output
```
(-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^(-1 + n)*(a + x*(b +
 c*x))])
```

3.121.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1962}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{3(n-1)}{2}}}{(ax^{n-1}+bx^n+cx^{n+1})^{3/2}}\,dx$$

$$\Big\downarrow \text{1962}$$

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

input
```
Int[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

output
```
(-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n) + b*x^n +
 c*x^(1 + n)])
```

3.121.3.1 Defintions of rubi rules used

rule 1962
```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[-2*x^((n - 1)/2)*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, 3
*((n - 1)/2)] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

3.121.4 Maple [F]

$$\int \frac{x^{-\frac{3}{2}+\frac{3n}{2}}}{\left(a\,x^{-1+n}+b\,x^{n}+c\,x^{1+n}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

output
```
int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = -\frac{2\,(2\,cx^2+bx)\sqrt{\frac{(cx^2+bx+a)x^{n+1}}{x^2}}}{(ab^2-4\,a^2c+(b^2c-4\,ac^2)x^2+(b^3-4\,abc)x)x^{\frac{1}{2}\,n+\frac{1}{2}}}$$

input
```
integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="
fricas")
```

output
```
-2*(2*c*x^2 + b*x)*sqrt((c*x^2 + b*x + a)*x^(n + 1)/x^2)/((a*b^2 - 4*a^2*c
 + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^(1/2*n + 1/2))
```

3.121.6 Sympy [F]

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3n}{2}-\frac{3}{2}}}{(ax^{n-1}+bx^n+cx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)
```

output
```
Integral(x**(3*n/2 - 3/2)/(a*x**(n - 1) + b*x**n + c*x**(n + 1))**(3/2), x
)
```

3.121.7 Maxima [F]

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}\,n-\frac{3}{2}}}{(cx^{n+1}+ax^{n-1}+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="
maxima")
```

output
```
integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

3.121.8 Giac [F]

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{(cx^{n+1}+ax^{n-1}+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="
giac")
```

output
```
integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx = \int \frac{x^{\frac{3n}{2}-\frac{3}{2}}}{(b\,x^n+a\,x^{n-1}+c\,x^{n+1})^{3/2}}\,dx$$

input
```
int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2),x)
```

output
```
int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2), x)
```

3.122 $\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx$

3.122.1 Optimal result

Integrand size = 27, antiderivative size = 287

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2dx^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}} + \frac{2ex^4\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

output
```
2/3*d*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+2/7*e*x^4*Appell
F1(7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 11.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.83

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(7dx^2\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+3ex^4\,\text{AppellF1}\left(\right.\right.}{21\sqrt{x\left(a+bx^2+cx^4\right)}}$$

input
```
Integrate[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x^2*AppellF1
[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + 3*e*x^4*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[x*(a +
 b*x^2 + c*x^4)])
```

3.122.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2001, 1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx$$

$$\Big\downarrow \text{ 2001}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{x}\left(ex^2+d\right)}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{ 1674}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\int \left(\frac{ex^{5/2}}{\sqrt{cx^4+bx^2+a}}+\frac{d\sqrt{x}}{\sqrt{cx^4+bx^2+a}}\right)dx}{\sqrt{ax+bx^3+cx^5}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\frac{2dx^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^2+cx^4}}+\frac{2ex^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+\cdots}}{\cdots}\right.}{\sqrt{ax+bx^3+cx^5}}$$

input
```
Int[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*((2*d*x^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4,
 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(3*Sqrt[a + b*x^2 + c*x^4]) + (2*e*x^(7/2)*Sqrt[1 + (2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[a*x + b*x^3
+ c*x^5]
```

3.122.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2001
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Simp[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p)   Int[x^(m + j*p)*(A + B*x^(k
- j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B
, j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && P
osQ[k - j]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.122.4 Maple [F]

$$\int \frac{x(e\,x^2+d)}{\sqrt{c\,x^5+b\,x^3+ax}}dx$$

input
```
int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

output
```
int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

3.122.5 Fricas [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^5 + b*x^3 + a*x)*(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

3.122.6 Sympy [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x(d+ex^2)}{\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

input
```
integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

output
```
Integral(x*(d + e*x**2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

3.122.7 Maxima [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.122.8 Giac [F]

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

input
```
integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \int \frac{x\,(e\,x^2+d)}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

input
```
int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

output
```
int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.123 $\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$

3.123.1 Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \frac{x\sqrt{3-3x^2+x^4}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

input
```
Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

output
```
(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(
Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

3.123.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^6-3x^4+3x^2}}dx$$

$$\Big\downarrow \text{1951}$$

$$-\int \frac{1}{12-\frac{9x^2(2-x^2)^2}{x^6-3x^4+3x^2}}d\frac{3x(2-x^2)}{\sqrt{x^6-3x^4+3x^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

input
```
Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

output
```
-1/2*ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

3.123.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

3.123.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}\,x}{2\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}\right)}{6}$	34
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{3}+2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+2\sqrt{x^{6}-3x^{4}+3x^{2}}}{x^{3}}\right)}{6}$	53
default	$\frac{x\sqrt{x^{4}-3x^{2}+3}\,\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6\sqrt{x^{6}-3x^{4}+3x^{2}}}$	58

input
```
int(1/(x^6-3*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^{3}+2\,\sqrt{3}(x^{3}-2\,x)+2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}\left(\sqrt{3}+2\right)-6\,x}{x^{3}}\right)$$

input
```
integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x
^2)*(sqrt(3) + 2) - 6*x)/x^3)
```

3.123.6 Sympy [F]

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \int \frac{1}{\sqrt{x^6-3x^4+3x^2}}\,dx$$

input
```
integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

3.123.7 Maxima [F]

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \int \frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}\,dx$$

input
```
integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx = \int \frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}\,dx$$

input
```
int(1/(3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

output
```
int(1/(3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

3.124 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

3.124.1 Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx = \frac{x\sqrt{3-3x^2+x^4}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

input
```
Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

output
```
(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(
Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

3.124.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (x^4-3x^2+3)}}dx$$

$$\Big\downarrow \text{ 2093}$$

$$\int \frac{1}{\sqrt{x^6-3x^4+3x^2}}dx$$

$$\Big\downarrow \text{ 1951}$$

$$-\int \frac{1}{12-\frac{9x^2(2-x^2)^2}{x^6-3x^4+3x^2}}d\frac{3x(2-x^2)}{\sqrt{x^6-3x^4+3x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

input
```
Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

output
```
-1/2*ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

3.124.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.124.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}\,x}{2\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}\right)}{6}$	34
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{3}-2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+2\sqrt{x^{6}-3x^{4}+3x^{2}}}{x^{3}}\right)}{6}$	52
default	$\frac{\sqrt{x^{4}-3x^{2}+3}\,x\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}$	58

input
```
int(1/(x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^{3}+2\,\sqrt{3}(x^{3}-2\,x)+2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}(\sqrt{3}+2)-6\,x}{x^{3}}\right)$$

input
```
integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x
^2)*(sqrt(3) + 2) - 6*x)/x^3)
```

3.124.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx=\int \frac{1}{\sqrt{x^2\left(x^4-3x^2+3\right)}}\,dx$$

input
```
integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

3.124.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx=\int \frac{1}{\sqrt{(x^4-3\,x^2+3)x^2}}\,dx$$

input
```
integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}}\,dx = \int \frac{1}{\sqrt{x^2\ (x^4-3\,x^2+3)}}\,dx$$

input
```
int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

output
```
int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.125 $\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx$

3.125.1 Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx=\frac{x\sqrt{3-3x^2+x^4}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

input
```
Integrate[1/Sqrt[1 - (1 - x^2)^3],x]
```

output
```
(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(
Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

3.125.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}dx$$

$$\Big\downarrow \textbf{2093}$$

$$\int \frac{1}{\sqrt{x^6-3x^4+3x^2}}dx$$

$$\Big\downarrow \textbf{1951}$$

$$-\int \frac{1}{12-\frac{9x^2(2-x^2)^2}{x^6-3x^4+3x^2}}d\frac{3x(2-x^2)}{\sqrt{x^6-3x^4+3x^2}}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

input
```
Int[1/Sqrt[1 - (1 - x^2)^3],x]
```

output
```
-1/2*ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

3.125.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.125.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}\,x}{2\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}\right)}{6}$	34
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{3}-2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+2\sqrt{x^{6}-3x^{4}+3x^{2}}}{x^{3}}\right)}{6}$	52
default	$\frac{x\sqrt{x^{4}-3x^{2}+3}\,\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6\sqrt{x^{6}-3x^{4}+3x^{2}}}$	58

input
```
int(1/(1-(-x^2+1)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^{3}+2\,\sqrt{3}(x^{3}-2\,x)+2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}\left(\sqrt{3}+2\right)-6\,x}{x^{3}}\right)$$

input
```
integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x
^2)*(sqrt(3) + 2) - 6*x)/x^3)
```

3.125.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx=\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx$$

input
```
integrate(1/(1-(-x**2+1)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - (1 - x**2)**3), x)
```

3.125.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx=\int \frac{1}{\sqrt{(x^2-1)^3+1}}\,dx$$

input
```
integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \int \frac{1}{\sqrt{(x^2-1)^3+1}}\,dx$$

input
```
int(1/((x^2 - 1)^3 + 1)^(1/2),x)
```

output
```
int(1/((x^2 - 1)^3 + 1)^(1/2), x)
```

3.126 $\int \sqrt{3x^2-3x^4+x^6}\,dx$

3.126.1 Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \sqrt{3x^2-3x^4+x^6}\,dx=-\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\operatorname{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

output
```
-1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1
/2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \sqrt{3x^2-3x^4+x^6}\,dx =\frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2(3-3x^2+x^4)}}$$

input
```
Integrate[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

output
```
(x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2
+ 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

3.126.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1950, 1432, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^6-3x^4+3x^2}\,dx$$

$$\Big\downarrow \text{1950}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\int x\sqrt{x^4-3x^2+3}dx}{x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\int \sqrt{x^4-3x^2+3}dx^2}{2x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\left(\frac{3}{8}\int \frac{1}{\sqrt{x^4-3x^2+3}}dx^2-\frac{1}{4}\left(3-2x^2\right)\sqrt{x^4-3x^2+3}\right)}{2x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\left(\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{x^4}{3}+1}}d\left(2x^2-3\right)-\frac{1}{4}\left(3-2x^2\right)\sqrt{x^4-3x^2+3}\right)}{2x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\left(\frac{3}{8}\text{arcsinh}\left(\frac{2x^2-3}{\sqrt{3}}\right)-\frac{1}{4}\left(3-2x^2\right)\sqrt{x^4-3x^2+3}\right)}{2x\sqrt{x^4-3x^2+3}}$$

input
```
Int[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

output
```
(Sqrt[3*x^2 - 3*x^4 + x^6]*(-1/4*((3 - 2*x^2)*Sqrt[3 - 3*x^2 + x^4]) + (3*
ArcSinh[(-3 + 2*x^2)/Sqrt[3]])/8))/(2*x*Sqrt[3 - 3*x^2 + x^4])
```

3.126.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1950
```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Simp[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))])   Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))],
x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

3.126.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{4\sqrt{x^2(x^4-3x^2+3)}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)x-6\sqrt{x^2(x^4-3x^2+3)}}{16x}$	62
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}-\frac{3\ln\left(\frac{-2x^3+2\sqrt{x^6-3x^4+3x^2}+3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^6-3x^4+3x^2}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

input
```
int((x^6-3*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/x*(4*(x^2*(x^4-3*x^2+3))^(1/2)*x^2+3*arcsinh(1/3*3^(1/2)*(2*x^2-3))*x
-6*(x^2*(x^4-3*x^2+3))^(1/2))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = -\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}(2\,x^2-3)-9\,x}{64\,x}$$

input
```
integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x
^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

3.126.6 Sympy [F]

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \int \sqrt{x^6-3x^4+3x^2}\,dx$$

input
```
integrate((x**6-3*x**4+3*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

3.126.7 Maxima [F]

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \int \sqrt{x^6-3\,x^4+3\,x^2}\,dx$$

input
```
integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\begin{aligned}&\int \sqrt{3x^2-3x^4+x^6}\,dx\\&=\frac{1}{16}\left(2\sqrt{x^4-3\,x^2+3}(2\,x^2-3)-3\,\log\left(-2\,x^2+2\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sgn}(x)\\&\quad+\frac{3}{16}\left(2\sqrt{3}+\log\left(2\sqrt{3}+3\right)\right)\mathrm{sgn}(x)\end{aligned}$$

input
```
integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*
x^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \int \sqrt{x^6-3\,x^4+3\,x^2}\,dx$$

input
```
int((3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

output
```
int((3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

3.127 $\int \sqrt{x^2 (3-3x^2+x^4)}\,dx$

3.127.1 Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \sqrt{x^2 (3-3x^2+x^4)}\,dx = -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\operatorname{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

output
```
-1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1
/2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \sqrt{x^2 (3-3x^2+x^4)}\,dx = \frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2(3-3x^2+x^4)}}$$

input
```
Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

output
```
(x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2
+ 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

3.127.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2093, 1950, 1432, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2 \left(x^4-3 x^2+3\right)} \, dx$$

$$\Big\downarrow \text{ 2093}$$

$$\int \sqrt{x^6-3 x^4+3 x^2}dx$$

$$\Big\downarrow \text{ 1950}$$

$$\frac{\sqrt{x^6-3 x^4+3 x^2} \int x \sqrt{x^4-3 x^2+3}dx}{x \sqrt{x^4-3 x^2+3}}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{\sqrt{x^6-3 x^4+3 x^2} \int \sqrt{x^4-3 x^2+3}dx^2}{2 x \sqrt{x^4-3 x^2+3}}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\sqrt{x^6-3 x^4+3 x^2} \left(\frac{3}{8} \int \frac{1}{\sqrt{x^4-3 x^2+3}}dx^2-\frac{1}{4} \left(3-2 x^2\right) \sqrt{x^4-3 x^2+3}\right)}{2 x \sqrt{x^4-3 x^2+3}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{\sqrt{x^6-3 x^4+3 x^2} \left(\frac{1}{8} \sqrt{3} \int \frac{1}{\sqrt{\frac{x^4}{3}+1}}d\left(2 x^2-3\right)-\frac{1}{4} \left(3-2 x^2\right) \sqrt{x^4-3 x^2+3}\right)}{2 x \sqrt{x^4-3 x^2+3}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\sqrt{x^6-3 x^4+3 x^2} \left(\frac{3}{8} \text{arcsinh}\left(\frac{2 x^2-3}{\sqrt{3}}\right)-\frac{1}{4} \left(3-2 x^2\right) \sqrt{x^4-3 x^2+3}\right)}{2 x \sqrt{x^4-3 x^2+3}}$$

input
```
Int[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

output
```
(Sqrt[3*x^2 - 3*x^4 + x^6]*(-1/4*((3 - 2*x^2)*Sqrt[3 - 3*x^2 + x^4]) + (3*
ArcSinh[(-3 + 2*x^2)/Sqrt[3]])/8))/(2*x*Sqrt[3 - 3*x^2 + x^4])
```

3.127.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1950
```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Simp[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))])   Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))],
x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.127.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{4\sqrt{x^2(x^4-3x^2+3)}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)x-6\sqrt{x^2(x^4-3x^2+3)}}{16x}$	62
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}+\frac{3\ln\left(\frac{2x^3+2\sqrt{x^6-3x^4+3x^2}-3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^2(x^4-3x^2+3)}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

input
```
int((x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/x*(4*(x^2*(x^4-3*x^2+3))^(1/2)*x^2+3*arcsinh(1/3*3^(1/2)*(2*x^2-3))*x
-6*(x^2*(x^4-3*x^2+3))^(1/2))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \sqrt{x^2\left(3-3x^2+x^4\right)}\,dx$$
$$= -\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}(2\,x^2-3)-9\,x}{64\,x}$$

input
```
integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
-1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x
^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

3.127.6 Sympy [F]

$$\int \sqrt{x^2 (3-3x^2+x^4)}\,dx = \int \sqrt{x^2 (x^4-3x^2+3)}\,dx$$

input
```
integrate((x**2*(x**4-3*x**2+3))**(1/2),x)
```

output
```
Integral(sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

3.127.7 Maxima [F]

$$\int \sqrt{x^2 (3-3x^2+x^4)}\,dx = \int \sqrt{(x^4-3\,x^2+3)x^2}\,dx$$

input
```
integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\begin{aligned}
&\int \sqrt{x^2 (3-3x^2+x^4)}\,dx \\
&= \frac{1}{16}\left(2\sqrt{x^4-3\,x^2+3}(2\,x^2-3)-3\,\log\left(-2\,x^2+2\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sgn}(x) \\
&\quad + \frac{3}{16}\left(2\sqrt{3}+\log\left(2\sqrt{3}+3\right)\right)\mathrm{sgn}(x)
\end{aligned}$$

input
```
integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*
x^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^2 (3 - 3x^2 + x^4)}\, dx = \int \sqrt{x^2 \,(x^4 - 3\,x^2 + 3)}\, dx$$

input
```
int((x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

output
```
int((x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.128 $\int \sqrt{1-\left(1-x^2\right)^3}\,dx$

3.128.1 Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \sqrt{1-\left(1-x^2\right)^3}\,dx=-\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\operatorname{arcsinh}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

output
```
-1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1
/2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \sqrt{1-\left(1-x^2\right)^3}\,dx =\frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2\left(3-3x^2+x^4\right)}}$$

input
```
Integrate[Sqrt[1 - (1 - x^2)^3],x]
```

output
```
(x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2
+ 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

3.128.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2093, 1950, 1432, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\left(1-x^2\right)^3}\,dx$$

$$\Big\downarrow \text{ 2093}$$

$$\int \sqrt{x^6-3x^4+3x^2}dx$$

$$\Big\downarrow \text{ 1950}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\int x\sqrt{x^4-3x^2+3}dx}{x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\int \sqrt{x^4-3x^2+3}dx^2}{2x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\left(\frac{3}{8}\int \frac{1}{\sqrt{x^4-3x^2+3}}dx^2-\frac{1}{4}\left(3-2x^2\right)\sqrt{x^4-3x^2+3}\right)}{2x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\left(\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{x^4}{3}+1}}d\left(2x^2-3\right)-\frac{1}{4}\left(3-2x^2\right)\sqrt{x^4-3x^2+3}\right)}{2x\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\sqrt{x^6-3x^4+3x^2}\left(\frac{3}{8}\text{arcsinh}\left(\frac{2x^2-3}{\sqrt{3}}\right)-\frac{1}{4}\left(3-2x^2\right)\sqrt{x^4-3x^2+3}\right)}{2x\sqrt{x^4-3x^2+3}}$$

input
```
Int[Sqrt[1 - (1 - x^2)^3],x]
```

output
```
(Sqrt[3*x^2 - 3*x^4 + x^6]*(-1/4*((3 - 2*x^2)*Sqrt[3 - 3*x^2 + x^4]) + (3*
ArcSinh[(-3 + 2*x^2)/Sqrt[3]])/8))/(2*x*Sqrt[3 - 3*x^2 + x^4])
```

3.128. $\int \sqrt{1-\left(1-x^2\right)^3}\,dx$

3.128.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1950
```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Simp[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))])   Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))],
x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.128.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{4\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}\, x^{2}+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,\left(2x^{2}-3\right)}{3}\right) x-6\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}{16x}$	62
trager	$\frac{\left(2x^{2}-3\right)\sqrt{x^{6}-3x^{4}+3x^{2}}}{8x}-\frac{3\ln\left(\frac{-2x^{3}+2\sqrt{x^{6}-3x^{4}+3x^{2}}+3x}{x}\right)}{16}$	64
risch	$\frac{\left(2x^{2}-3\right)\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}{8x}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x^{2}-\frac{3}{2}\right)}{3}\right)\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}{16\sqrt{x^{4}-3x^{2}+3}\, x}$	74
default	$\frac{\sqrt{x^{6}-3x^{4}+3x^{2}}\,\left(4\sqrt{x^{4}-3x^{2}+3}\, x^{2}+3\,\operatorname{arcsinh}\left(\frac{\sqrt{3}\,\left(2x^{2}-3\right)}{3}\right)-6\sqrt{x^{4}-3x^{2}+3}\right)}{16x\sqrt{x^{4}-3x^{2}+3}}$	81

input
```
int((1-(-x^2+1)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/x*(4*(x^2*(x^4-3*x^2+3))^(1/2)*x^2+3*arcsinh(1/3*3^(1/2)*(2*x^2-3))*x
-6*(x^2*(x^4-3*x^2+3))^(1/2))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \sqrt{1-\left(1-x^{2}\right)^{3}}\, dx = -\frac{12\, x\log\left(-\frac{2\, x^{3}-3\, x-2\, \sqrt{x^{6}-3\, x^{4}+3\, x^{2}}}{x}\right)-8\, \sqrt{x^{6}-3\, x^{4}+3\, x^{2}}(2\, x^{2}-3)-9\, x}{64\, x}$$

input
```
integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x
^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

3.128.6 Sympy [F]

$$\int \sqrt{1-\left(1-x^2\right)^3}\,dx=\int \sqrt{1-\left(1-x^2\right)^3}\,dx$$

input
```
integrate((1-(-x**2+1)**3)**(1/2),x)
```

output
```
Integral(sqrt(1 - (1 - x**2)**3), x)
```

3.128.7 Maxima [F]

$$\int \sqrt{1-\left(1-x^2\right)^3}\,dx=\int \sqrt{\left(x^2-1\right)^3+1}\,dx$$

input
```
integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x^2 - 1)^3 + 1), x)
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\begin{aligned}&\int \sqrt{1-\left(1-x^2\right)^3}\,dx\\&=\frac{1}{16}\left(2\sqrt{x^4-3x^2+3}(2x^2-3)-3\log\left(-2x^2+2\sqrt{x^4-3x^2+3}+3\right)\right)\mathrm{sgn}(x)\\&\quad+\frac{3}{16}\left(2\sqrt{3}+\log\left(2\sqrt{3}+3\right)\right)\mathrm{sgn}(x)\end{aligned}$$

input
```
integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*
x^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-(1-x^2)^3}\,dx = \int \sqrt{(x^2-1)^3+1}\,dx$$

input
```
int(((x^2 - 1)^3 + 1)^(1/2),x)
```

output
```
int(((x^2 - 1)^3 + 1)^(1/2), x)
```

3.129 $\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$

3.129.1 Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a]
```

3.129.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

3.129.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.129.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{a}}$	35

input
```
int(1/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.92

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \left[\frac{\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)}{a}\right]$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]
```

3.129.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x + c*x**2)), x)
```

3.129.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
-log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x)/a^(1/2)
```

3.130 $\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$

3.130.1 Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx = \frac{2x\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

output
```
(2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqr
t[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

3.130.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (a+bx+cx^2)}}dx$$

$$\Big\downarrow \text{2093}$$

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{1951}$$

$$-2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

output
```
-(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a]
)
```

3.130.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.130.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)}{\sqrt{a}}$	42
default	$-\frac{x\sqrt{c\,x^{2}+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{x^{2}(c\,x^{2}+bx+a)}\,\sqrt{a}}$	64

input
```
int(1/(x^2*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(ln(2)-ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(1/2)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = \left[\frac{\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

input
```
integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

3.130.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = \int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

input
```
integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.130.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = \int \frac{1}{\sqrt{(cx^2+bx+a)x^2}}\,dx$$

input
```
integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = -\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2 (a+bx+cx^2)}}\, dx = \int \frac{1}{\sqrt{x^2\, \left(c\, x^2+b\, x+a\right)}}\, dx$$

input
```
int(1/(x^2*(a + b*x + c*x^2))^(1/2),x)
```

output
```
int(1/(x^2*(a + b*x + c*x^2))^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx$

3.131.1 Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*(b*x+2*a)*x^(1/2)/a^(1/2)/(c*x^3+b*x^2+a*x)^(1/2))/a^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx=\frac{2\sqrt{x}\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x(a+x(b+cx))}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

output
```
(2*Sqrt[x]*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)
])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + x*(b + c*x))])
```

3.131.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2035, 2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}dx$$

$$\Big\downarrow \text{ 2035}$$

$$2\int \frac{1}{\sqrt{x\left(cx^2+bx+a\right)}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2093}$$

$$2\int \frac{1}{\sqrt{cx^3+bx^2+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1951}$$

$$-2\int \frac{1}{4a-x}d\frac{\sqrt{x}(2a+bx)}{\sqrt{cx^3+bx^2+ax}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

output
```
-(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqr
t[a])
```

3.131.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.131.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{c\,x^{2}+b x+a}\,\ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{\sqrt{x\left(c\,x^{2}+b x+a\right)}\,\sqrt{a}}$	64

input
```
int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2
*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx$$
$$=\left[\frac{\log\left(\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{a}\sqrt{x}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

input
```
integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 +
 a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(
c*x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2
*x))/a]
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.131.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx = \int \frac{1}{\sqrt{(cx^2+bx+a)x}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx=\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - 2*arcta
n(sqrt(a)/sqrt(-a))/sqrt(-a)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

input
```
int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)), x)
```

3.132 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx$

3.132.1 Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*x^(3/2)*(b*x+2*a)/a^(1/2)/(c*x^5+b*x^4+a*x^3)^(1/2))/a^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx = \frac{2x^{3/2}\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

output
```
(2*x^(3/2)*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)
])/Sqrt[a]])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))])
```

3.132.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2035, 2094, 1960, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a+bx+cx^2)}} dx$$

$$\Big\downarrow \text{ 2035}$$

$$2\int \frac{x}{\sqrt{x^3 (cx^2+bx+a)}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2094}$$

$$2\int \frac{x}{\sqrt{cx^5+bx^4+ax^3}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1960}$$

$$-2\int \frac{1}{4a-x}d\frac{x^{3/2}(2a+bx)}{\sqrt{cx^5+bx^4+ax^3}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

input
```
Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

output
```
-(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/S
qrt[a])
```

3.132.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1960
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[-2/(n - q)   Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*
((2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m
, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[m, q/2 - 1]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2094
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

3.132.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{x^{\frac{3}{2}} \sqrt{c\,x^{2}+b x +a}\, \ln\left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right)}{\sqrt{x^{3} \left(c\,x^{2}+b x +a \right)}\, \sqrt{a}}$	66

input
```
int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx$$
$$=\left[\frac{\log\left(\frac{8\,abx^3+\left(b^2+4\,ac\right)x^4+8\,a^2x^2-4\,\sqrt{cx^5+bx^4+ax^3}(bx+2\,a)\sqrt{a}\sqrt{x}}{x^4}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^4+ax^3}(bx+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^4+abx^3+a^2x^2)}\right)}{a}\right]$$

input
```
integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4
 + a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*s
qrt(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3
 + a^2*x^2))/a]
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx=\text{Timed out}$$

input
```
integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx=\int \frac{\sqrt{x}}{\sqrt{(cx^2+bx+a)x^3}}\,dx$$

input
```
integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx = \frac{2\left(\frac{\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2*(arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - arctan
(sqrt(a)/sqrt(-a))/sqrt(-a))/sgn(x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^3\,(c\,x^2+b\,x+a)}}\,dx$$

input
```
int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2),x)
```

output
```
int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2), x)
```

3.133 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

3.133.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a]
```

3.133.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1434, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1154}$$

$$-\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

3.133.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.133.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39
elliptic	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39
pseudoelliptic	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39

input
```
int(1/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)}{2\,a}\right]$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^
2 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

3.133.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

3.133.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx=\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}} - \frac{\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\sqrt{a}}$$

input

```
int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output

```
- log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
 b*x^2)/(2*a^(1/2))
```

3.134 $\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx$

3.134.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh(1/2*x*(b*x^2+2*a)/a^(1/2)/(c*x^6+b*x^4+a*x^2)^(1/2))/a^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx=\frac{x\sqrt{a+bx^2+cx^4}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2\left(a+bx^2+cx^4\right)}}$$

input
```
Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

output
```
(x*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])
/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c*x^4)])
```

3.134.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (a+bx^2+cx^4)}}dx$$

$$\Big\downarrow \text{ 2093}$$

$$\int \frac{1}{\sqrt{ax^2+bx^4+cx^6}}dx$$

$$\Big\downarrow \text{ 1951}$$

$$-\int \frac{1}{4a-\frac{x^2(bx^2+2a)^2}{cx^6+bx^4+ax^2}}d\frac{x(bx^2+2a)}{\sqrt{cx^6+bx^4+ax^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

input
```
Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

output
```
-1/2*ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/Sq
rt[a]
```

3.134.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.134.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{x\sqrt{c\,x^{4}+b\,x^{2}+a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{x^{2}\left(c\,x^{4}+b\,x^{2}+a\right)}\,\sqrt{a}}$	72

input
```
int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2*(c*x^4+b*x^2+a))^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*a+b
*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^6+bx^4+ax^2}\,(bx^2+2\,a)\sqrt{a}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^4+ax^2}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a}\right]$$

input
```
integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^6 + b*x^4
+ a*x^2)*(b*x^2 + 2*a)*sqrt(a))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt
(c*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^5 + a*b*x^3 + a^2*x)
)/a]
```

3.134.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx$$

input
```
integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(a + b*x**2 + c*x**4)), x)
```

3.134.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{(cx^4+bx^2+a)x^2}}\,dx$$

input
```
integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

output
```
-arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + arctan(-(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx=\int \frac{1}{\sqrt{x^2\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2),x)
```

output
```
int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2), x)
```

3.135 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx$

3.135.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1
/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x(a+bx^2+cx^4)}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c
*x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

3.135.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2035, 2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}dx$$

$$\Big\downarrow \text{2035}$$

$$2\int \frac{1}{\sqrt{x\left(cx^4+bx^2+a\right)}}d\sqrt{x}$$

$$\Big\downarrow \text{2093}$$

$$2\int \frac{1}{\sqrt{cx^5+bx^3+ax}}d\sqrt{x}$$

$$\Big\downarrow \text{1951}$$

$$-\int \frac{1}{4a-x}d\frac{\sqrt{x}\left(bx^2+2a\right)}{\sqrt{cx^5+bx^3+ax}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])
]/Sqrt[a]
```

3.135.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.135.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\sqrt{a}}$	72

input
```
int(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2
*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.69

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx$$
$$= \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a}\right]$$

input
```
integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3
+ a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/
2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*
x^3 + a^2*x))/a]
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)
```

output
```
Timed out
```

3.135.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{(cx^4+bx^2+a)x}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

output
```
arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arcta
n(sqrt(a)/sqrt(-a))/sqrt(-a)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{x\,(c\,x^4+b\,x^2+a)}}\,dx$$

input
```
int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)), x)
```

3.136 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx$

3.136.1 Optimal result

Integrand size = 26, antiderivative size = 53

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x^{3/2}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh(1/2*x^(3/2)*(b*x^2+2*a)/a^(1/2)/(c*x^7+b*x^5+a*x^3)^(1/2))/a^
(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx=\frac{x^{3/2}\sqrt{a+bx^2+cx^4}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^3\left(a+bx^2+cx^4\right)}}$$

input
```
Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

output
```
(x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c
*x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x^3*(a + b*x^2 + c*x^4)])
```

3.136.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2035, 2094, 1960, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a+bx^2+cx^4)}}dx$$

$$\Big\downarrow \text{ 2035}$$

$$2\int \frac{x}{\sqrt{x^3 (cx^4+bx^2+a)}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2094}$$

$$2\int \frac{x}{\sqrt{cx^7+bx^5+ax^3}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1960}$$

$$-\int \frac{1}{4a-x}d\frac{x^{3/2} (bx^2+2a)}{\sqrt{cx^7+bx^5+ax^3}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{x^{3/2} (2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

input
```
Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

output
```
-1/2*ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7
])]/Sqrt[a]
```

3.136.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1960
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[-2/(n - q)   Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*
((2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m
, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[m, q/2 - 1]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2094
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

3.136.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

method	result	size
default	$-\frac{x^{\frac{3}{2}}\sqrt{c\,x^{4}+b\,x^{2}+a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{x^{3}\left(c\,x^{4}+b\,x^{2}+a\right)}\,\sqrt{a}}$	74

input
```
int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln(
(2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx$$
$$=\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^4+8\,a^2x^2-4\,\sqrt{cx^7+bx^5+ax^3}\,(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^6}\right)}{4\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^7+bx^5+ax^3}\,(bx^2+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^6+abx^4+a^2x^2)}\right)}{2\,a}\right]$$

input
```
integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^7 + b*x^
5 + a*x^3)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^6)/sqrt(a), 1/2*sqrt(-a)*arcta
n(1/2*sqrt(c*x^7 + b*x^5 + a*x^3)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^6
+ a*b*x^4 + a^2*x^2))/a]
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{(cx^4+bx^2+a)x^3}}\,dx$$

input
```
integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx = \frac{\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{\operatorname{sgn}(x)}$$

input
```
integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

output
```
(arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arct
an(sqrt(a)/sqrt(-a))/sqrt(-a))/sgn(x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^3\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

input
```
int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2),x)
```

output
```
int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2), x)
```

3.137 $\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx$

3.137.1 Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{3-3x^2+x^4}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/2*(-x^2+2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))*3^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

output
```
ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]]/Sqrt[3]
```

3.137.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^4-3x^2+3}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4-3x^2+3}}dx^2$$

$$\Big\downarrow \text{1154}$$

$$-\int \frac{1}{12-x^4}d\frac{3(2-x^2)}{\sqrt{x^4-3x^2+3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

input
```
Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/Sqrt[3]
```

3.137.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

3.137.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6}$	29
default	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-3x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}-3x^{2}+3}}\right)}{6}$	31
elliptic	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-3x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}-3x^{2}+3}}\right)}{6}$	31
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{2}+2\sqrt{x^{4}-3x^{2}+3}-2\operatorname{RootOf}\left(\_Z^{2}-3\right)}{x^{2}}\right)}{6}$	47

input
```
int(1/x/(x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^2+2\,\sqrt{3}(x^2-2)+2\,\sqrt{x^4-3\,x^2+3}(\sqrt{3}+2)-6}{x^2}\right)$$

input
```
integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(3*x^2 + 2*sqrt(3)*(x^2 - 2) + 2*sqrt(x^4 - 3*x^2 + 3)*(s
qrt(3) + 2) - 6)/x^2)
```

3.137.6 Sympy [F]

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \int \frac{1}{x\sqrt{x^4-3x^2+3}}\,dx$$

input
```
integrate(1/x/(x**4-3*x**2+3)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = -\frac{1}{6}\sqrt{3}\,\text{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

input
```
integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right) - \frac{1}{6}\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)$$

input
```
integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - 1/6*sqrt(3)*log(-
x^2 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3))
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = -\frac{\sqrt{3}\left(\ln\left(x^2-\frac{2\sqrt{3}\sqrt{x^4-3x^2+3}}{3}-2\right)+\ln\left(\frac{1}{x^2}\right)\right)}{6}$$

input
```
int(1/(x*(x^4 - 3*x^2 + 3)^(1/2)),x)
```

output
```
-(3^(1/2)*(log(x^2 - (2*3^(1/2)*(x^4 - 3*x^2 + 3)^(1/2))/3 - 2) + log(1/x^
2)))/6
```

3.138 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

3.138.1 Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx=\frac{x\sqrt{3-3x^2+x^4}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2\left(3-3x^2+x^4\right)}}$$

input
```
Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

output
```
(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(
Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

3.138.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (x^4-3x^2+3)}}dx$$
$$\Big\downarrow \text{ 2093}$$
$$\int \frac{1}{\sqrt{x^6-3x^4+3x^2}}dx$$
$$\Big\downarrow \text{ 1951}$$
$$-\int \frac{1}{12-\frac{9x^2(2-x^2)^2}{x^6-3x^4+3x^2}}d\frac{3x(2-x^2)}{\sqrt{x^6-3x^4+3x^2}}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

input
```
Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

output
```
-1/2*ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

3.138.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.138.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}\,x}{2\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}\right)}{6}$	34
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{3}+2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+2\sqrt{x^{6}-3x^{4}+3x^{2}}}{x^{3}}\right)}{6}$	53
default	$\frac{\sqrt{x^{4}-3x^{2}+3}\,x\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(x^{2}-2\right)\sqrt{3}}{2\sqrt{x^{4}-3x^{2}+3}}\right)}{6\sqrt{x^{2}\left(x^{4}-3x^{2}+3\right)}}$	58

input
```
int(1/(x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)*x/(x^2*(x^4-3*x^2+3))^(1/2))
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^{3}+2\,\sqrt{3}(x^{3}-2\,x)+2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}(\sqrt{3}+2)-6\,x}{x^{3}}\right)$$

input
```
integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x
^2)*(sqrt(3) + 2) - 6*x)/x^3)
```

3.138.6 Sympy [F]

$$\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}}\,dx = \int \frac{1}{\sqrt{x^2 (x^4-3x^2+3)}}\,dx$$

input
```
integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

3.138.7 Maxima [F]

$$\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}}\,dx = \int \frac{1}{\sqrt{(x^4-3\,x^2+3)x^2}}\,dx$$

input
```
integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}}\,dx = \frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3\,x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3\,x^2+3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}}\,dx=\int \frac{1}{\sqrt{x^2\,\left(x^4-3\,x^2+3\right)}}\,dx$$

input
```
int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

output
```
int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.139 $\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx$

3.139.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}}$$

output
```
-1/3*arctanh(1/2*(2-x)*3^(1/2)*x^(1/2)/(x^3-3*x^2+3*x)^(1/2))*3^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \frac{2\sqrt{x}\sqrt{3-3x+x^2}\operatorname{arctan}\left(\frac{x-\sqrt{3-3x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x(3-3x+x^2)}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

output
```
(2*Sqrt[x]*Sqrt[3 - 3*x + x^2]*ArcTanh[(x - Sqrt[3 - 3*x + x^2])/Sqrt[3]])
/(Sqrt[3]*Sqrt[x*(3 - 3*x + x^2)])
```

3.139.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2035, 2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x(x^2-3x+3)}}dx$$
$$\Big\downarrow \text{ 2035}$$
$$2\int \frac{1}{\sqrt{x(x^2-3x+3)}}d\sqrt{x}$$
$$\Big\downarrow \text{ 2093}$$
$$2\int \frac{1}{\sqrt{x^3-3x^2+3x}}d\sqrt{x}$$
$$\Big\downarrow \text{ 1951}$$
$$-2\int \frac{1}{12-x}d\frac{3(2-x)\sqrt{x}}{\sqrt{x^3-3x^2+3x}}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

output
```
-(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])
```

3.139.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

3.139.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{x^2-3x+3}\,\sqrt{3}\,\operatorname{arctanh}\left(\frac{(x-2)\sqrt{3}}{2\sqrt{x^2-3x+3}}\right)}{3\sqrt{x(x^2-3x+3)}}$	50

input
```
int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(1/2)/(x*(x^2-3*x+3))^(1/2)*(x^2-3*x+3)^(1/2)*3^(1/2)*arctanh(1/2*(x
-2)*3^(1/2)/(x^2-3*x+3)^(1/2))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{7\,x^3+4\,\sqrt{3}\sqrt{x^3-3\,x^2+3\,x}(x-2)\sqrt{x}-24\,x^2+24\,x}{x^3}\right)$$

input
```
integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((7*x^3 + 4*sqrt(3)*sqrt(x^3 - 3*x^2 + 3*x)*(x - 2)*sqrt(x)
 - 24*x^2 + 24*x)/x^3)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(3-3x+x^2\right)}}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(3-3x+x^2\right)}}\,dx=\int \frac{1}{\sqrt{(x^2-3\,x+3)x}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)), x)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(3-3x+x^2\right)}}\,dx=\frac{1}{3}\,\sqrt{3}\log\left(x+\sqrt{3}-\sqrt{x^2-3\,x+3}\right)-\frac{1}{3}\,\sqrt{3}\log\left(-x+\sqrt{3}+\sqrt{x^2-3\,x+3}\right)$$

input
```
integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*log(x + sqrt(3) - sqrt(x^2 - 3*x + 3)) - 1/3*sqrt(3)*log(-x +
sqrt(3) + sqrt(x^2 - 3*x + 3))
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(3-3x+x^2\right)}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(x^2-3\,x+3\right)}}\,dx$$

input
```
int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)), x)
```

3.140 $\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$

3.140.1 Optimal result

Integrand size = 36, antiderivative size = 70

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{\sqrt{a}(n-q)}$$

output
```
-arctanh(1/2*x^(1/2*q)*(2*a+b*x^(n-q))/a^(1/2)/(b*x^n+c*x^(2*n-q)+a*x^q)^(
1/2))/(n-q)/a^(1/2)
```

3.140.2 Mathematica [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx=\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$$

input
```
Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

output
```
Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]
```

3.140.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1960, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{q}{2}-1}}{\sqrt{ax^q+bx^n+cx^{2n-q}}}dx$$

$$\Big\downarrow \text{1960}$$

$$-\frac{2\int \frac{1}{4a-\frac{x^q\left(bx^{n-q}+2a\right)^2}{bx^n+cx^{2n-q}+ax^q}}d\frac{x^{q/2}\left(bx^{n-q}+2a\right)}{\sqrt{bx^n+cx^{2n-q}+ax^q}}}{n-q}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{x^{q/2}\left(2a+bx^{n-q}\right)}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

input
```
Int[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

output
```
-(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c*x^(2*n -
 q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

3.140.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1960
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[-2/(n - q)   Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*
((2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m
, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[m, q/2 - 1]
```

3.140.4 Maple [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{b\,x^{n}+c\,x^{2n-q}+a\,x^{q}}}dx$$

input
```
int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

output
```
int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

3.140.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^{n}+cx^{2n-q}+ax^{q}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.140.6 Sympy [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^{n}+cx^{2n-q}+ax^{q}}}\,dx = \int \frac{x^{\frac{q}{2}-1}}{\sqrt{ax^{q}+bx^{n}+cx^{2n-q}}}\,dx$$

input
```
integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
```

output
```
Integral(x**(q/2 - 1)/sqrt(a*x**q + b*x**n + c*x**(2*n - q)), x)
```

3.140.7 Maxima [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx=\int \frac{x^{\frac{1}{2}\,q-1}}{\sqrt{cx^{2\,n-q}+bx^n+ax^q}}\,dx$$

input
```
integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

3.140.8 Giac [F]

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx=\int \frac{x^{\frac{1}{2}\,q-1}}{\sqrt{cx^{2\,n-q}+bx^n+ax^q}}\,dx$$

input
```
integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx=\int \frac{x^{\frac{q}{2}-1}}{\sqrt{b\,x^n+a\,x^q+c\,x^{2\,n-q}}}\,dx$$

input
```
int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2),x)
```

output
```
int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
                finalresult={"A"," "}
            ,(*ELSE*)
                finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
        else:
            return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])  #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```